# Verification — X-Braced Bay with Tension-Only Diagonals

*FEM Calculator nonlinear (active-set) solver checked against statics, the method of joints and the force method*

| | |
|---|---|
| **Structure type** | Pin-jointed portal bay braced by two crossing tension-only rods, horizontal load at the top in either direction |
| **Verification method** | Nonlinear: the compressed rod goes slack, leaving a statically determinate truss — method of joints and virtual work. Linear (both rods acting): force method with one redundant |
| **Quantities checked** | Support reactions, axial force in every member (including the slack rod), node deflections; load reversal; a combination solved on its total load |
| **Result** | **Exact match** — FEM agrees with the hand solution to ≈$10^{-14}$ (floating-point round-off only) in every state |

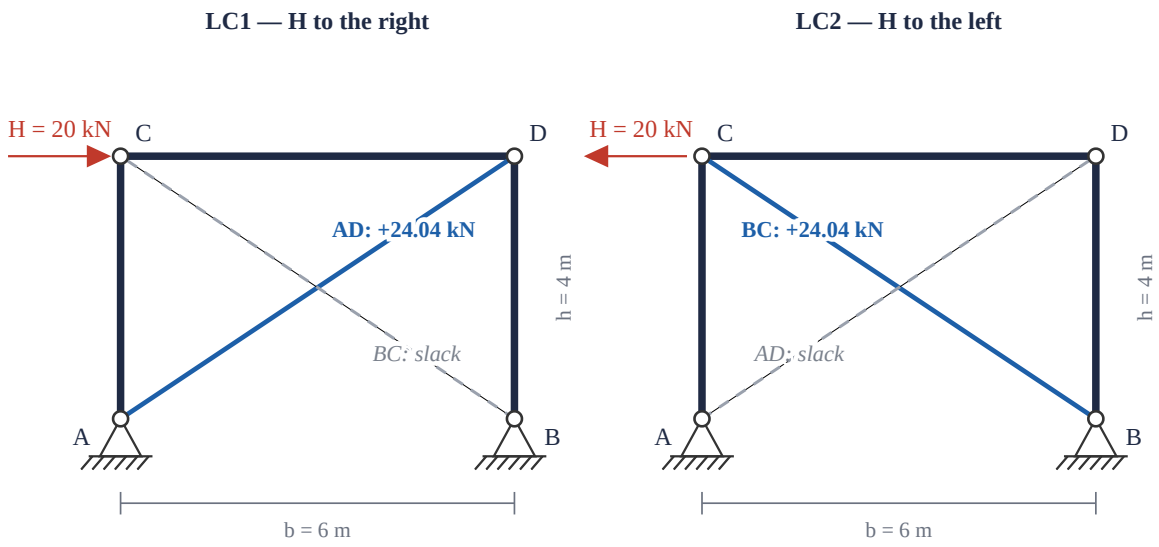


*Fig. 1 — The braced bay under both load directions. The rod that would be compressed goes slack (dashed); the other one carries the whole horizontal load.*

## 1. Structure and loading

A single braced bay, b = 6 m wide and h = 4 m high: two columns AC and BD and a beam CD, all pin-jointed (axial-only members, HEA 160, S235, E = 210 GPa, $EA_c$ = 814 438 kN), on pinned supports A and B. The bay is braced by two crossing round bars Ø 20 ($EA_d$ = 65 973 kN, $L_d = \sqrt{(b^2 + h^2)}$ = 7.2111 m) defined as **tension-only** members: a slender rod buckles at a negligible compressive force, so it is

designed to carry tension only and is assumed to go slack in compression. A horizontal load H = 20 kN acts at C, to the right in LC1 and to the left in LC2.

This is the most common nonlinearity in steel structures. Which rod works is not known in advance — it depends on the load — so the solver has to find it (the *active set*), and results of different load cases can no longer be added together. Both points are checked below.

## 2. Hand solution

Data: H = 20 kN, b = 6 m, h = 4 m, E = 210 GPa = $210 \cdot 10^6$ kN/m². Columns and beam HEA 160: $A_c$ = 38.7828 cm² → $EA_c = 210 \cdot 10^6$ kN/m² · $3.878277 \cdot 10^{-3}$ m² = **814 438.1 kN**. Rods Ø 20: $A_d = \pi \cdot (0.020\ \text{m})^2/4$ = 3.1416 cm² → $EA_d = 210 \cdot 10^6$ kN/m² · $3.141593 \cdot 10^{-4}$ m² = **65 973.4 kN**. Diagonal $L_d = \sqrt{b^2 + h^2} = \sqrt{(6\ \text{m})^2 + (4\ \text{m})^2}$ = **7.211103 m**; $b/L_d$ = 0.832050, $h/L_d$ = 0.554700. Tension positive.

**2.1 Nonlinear: one rod active.** Under LC1 the top of the bay moves right, which lengthens AD and shortens BC, so BC goes slack. The remaining four bars and four support reactions make a statically determinate truss. Method of joints:

> Joint C (H, column AC, beam CD): $\Sigma F_x$: $N_{CD} + H = 0 \rightarrow N_{CD} = -H =$ **−20 kN**; $\Sigma F_z$: $N_{AC} =$ **0**
>
> Joint D: $\Sigma F_x$: $N_{AD} \cdot b/L_d = H \rightarrow N_{AD} = H \cdot L_d/b$ = 20 kN · 7.211103 m / 6 m = **24.037009 kN**
>
> $\Sigma F_z$: $N_{BD} = -N_{AD} \cdot h/L_d = -H \cdot h/b$ = −20 kN · 4 m / 6 m = **−13.333333 kN** (2)(1)
>
> Reactions: $A_x = -H =$ **−20 kN**, $A_z = -H \cdot h/b =$ **−13.333333 kN** (A is held down), $B_x =$ **0**, $B_z = +H \cdot h/b =$ **13.333333 kN**
>
> Sway at C by virtual work (the unit load at C gives n = N/H): $\delta_C = \Sigma\ N^2 \cdot L/(H \cdot EA)$
>
> CD (beam): (−20.000000 kN)² · 6.000000 m / (20 kN · 814 438.1 kN) = 0.147341 mm
>
> BD (column): (−13.333333 kN)² · 4.000000 m / (20 kN · 814 438.1 kN) = 0.043657 mm
>
> AD (diagonal): (24.037009 kN)² · 7.211103 m / (20 kN · 65 973.4 kN) = 3.157645 mm
>
> $\delta_C$ = 0.147341 + 0.043657 + 3.157645 = **3.348643 mm** (3)
>
> Sway at D: $\delta_D = \delta_C - H \cdot b/EA_c$ = 3.348643 mm − 20 kN · 6 m / 814 438.1 kN · 1000 mm/m = **3.201302 mm**; vertical at D: $N_{BD} \cdot h/EA_c$ = −13.333333 kN · 4 m / 814 438.1 kN · 1000 mm/m = **−0.065485 mm**

Under LC2 the picture mirrors: AD goes slack and BC carries the same 24.0370 kN. The load path is different, though — joint D now has no active diagonal, so the beam and column BD carry nothing, column AC takes $-H \cdot h/b$, and the beam does not shorten, so C and D sway together:

Reactions: $A_x$ = **0**, $A_z$ = +H·h/b = **13.333333 kN**, $B_x$ = +H = **20 kN**, $B_z$ = −H·h/b = **−13.333333 kN**; $N_{BC}$ = **24.037009 kN**, $N_{AC}$ = **−13.333333 kN**

$\delta_C = \delta_D = H\cdot[(h/b)^2\cdot h/EA_c + (L_d/b)^2\cdot L_d/EA_d]$ = 20 kN · [(0.666667)² · 4 m / 814 438.1 kN + (1.201850)² · 7.211103 m / 65 973.4 kN] = **3.201302 mm** (to the left) (4)

**2.2 Linear: both rods active.** If the rods could take compression (ordinary truss members — the result a linear analysis gives), the bay has one redundant. Take X = $N_{BC}$. The determinate state $N_0$ is the LC1 solution above; the unit state $n_1$ (X = 1 in BC) follows from the same joints: $n_1$ = 1 in AD and BC, −b/$L_d$ in CD, −h/$L_d$ in both columns. Every term of the flexibility sums:

| Member | L [m] | EA [kN] | $N_0$ [kN] | $n_1$ | $N_0\cdot n_1\cdot L/EA$ [$10^{-6}$ m] | $n_1^2\cdot L/EA$ [$10^{-6}$ m/kN] | $N = N_0 + X\cdot n_1$ [kN] | $N\cdot n_0\cdot L/EA$ [$10^{-6}$ m] |
|---|---|---|---|---|---|---|---|---|
| AD | 7.2111 | 65 973.4 | 24.0370 | 1.000000 | 2627.319695 | 109.303106 | 11.7481 | 1543.307068 |
| BC | 7.2111 | 65 973.4 | 0.0000 | 1.000000 | 0.000000 | 109.303106 | −12.2889 | 0.000000 |
| AC | 4.0000 | 814 438.1 | 0.0000 | −0.554700 | 0.000000 | 1.511188 | 6.8166 | 0.000000 |
| BD | 4.0000 | 814 438.1 | −13.3333 | −0.554700 | 36.324444 | 1.511188 | −6.5167 | 21.337248 |
| CD | 6.0000 | 814 438.1 | −20.0000 | −0.832050 | 122.594998 | 5.100260 | −9.7751 | 72.013211 |
| **Σ** | | | | | **2786.239137** | **226.728849** | | **1636.657526** |

$X = -\delta_{10}/\delta_{11}$ = −(2786.239137·$10^{-6}$ m) / (226.728849·$10^{-6}$ m/kN) = **−12.288860 kN** (5)

$N = N_0 + X\cdot n_1$, e.g. $N_{AD}$ = 24.037009 kN + (−12.288860 kN) · 1 = **11.748148 kN**, $N_{CD}$ = −20 kN + (−12.288860 kN) · (−0.832050) = **−9.775050 kN** (all members in the table)

Reactions: $A_x = -N_{AD}\cdot b/L_d$ = −11.748148 kN · 0.832050 = **−9.775050 kN**; $B_x = N_{BC}\cdot b/L_d$ = −12.288860 kN · 0.832050 = **−10.224950 kN**

$A_z = -(N_{AC} + N_{AD}\cdot h/L_d)$ = −(6.816633 + (11.748148) · 0.554700) kN = **−13.333333 kN**;

$B_z = -(N_{BD} + N_{BC}\cdot h/L_d)$ = −((−6.516700) + (−12.288860) · 0.554700) kN = **13.333333 kN**

Sway: $\delta_C = \Sigma\ N\cdot n_0\cdot L/EA$ = 1636.657526·$10^{-6}$ m = **1.636658 mm**

Each rod takes roughly half the horizontal load — not exactly half, because the columns stretch and shorten too — and the bay is about twice as stiff. In reality the compressed Ø 20 rod (L = 7.2 m) would buckle long before 12 kN, which is exactly why tension-only bracing is analysed nonlinearly.

**2.3 Combination CO1 = 1.0·LC1 + 0.5·LC2.** Net load $H_{net}$ = 1.0 · 20 kN − 0.5 · 20 kN = 10 kN to the right, so LC1's active set at half the load: $N_{AD}$ = 24.037009 kN / 2 = **12.018504 kN**, $N_{BC}$ = 0, $\delta_C$ = 3.348643 mm / 2 = **1.674321 mm**.

## 3. FEM model

The structure is stored as a regular project file, `x-braced-bay.json`: 4 nodes, 5 members, the two diagonals with element type *tension-only truss*, the columns and beam ordinary truss members, out-of-plane translation restrained at every node. Two load cases (LC1, LC2) and one combination CO1 = 1.0·LC1 + 0.5·LC2. The model is solved with the calculator's core solver (`analyze()`), which detects the tension-only members and runs the active-set iteration. The linear results of section 2.2 are the same model with the two diagonals changed to ordinary truss members — the same state the solver keeps internally as its superposition basis.

## 4. Results comparison

**4.1 LC1, nonlinear** (H to the right — AD active, BC slack)

| Quantity | Formula | Hand solution | FEM result | Difference |
|---|---|---|---|---|
| **Support reactions [kN]** | | | | |
| $A_x$ | −H | −20.000000 | −19.999999999999996 | ≈ 0 |
| $A_z$ | −H·h/b | −13.333333 | −13.333333333333330 | ≈ 0 |
| $B_x$ | 0 | 0.000000 | 0.000000000000000 | 0 |
| $B_z$ | +H·h/b | 13.333333 | 13.333333333333330 | ≈ 0 |
| **Axial forces [kN] (tension +)** | | | | |
| Diagonal AD (active) | $H \cdot L_d/b$ | 24.037009 | 24.037008503093261 | 0 |
| Diagonal BC (slack) | 0 | 0.000000 | 0.000000000000000 | 0 |
| Column AC | 0 | 0.000000 | 0.000000000000000 | 0 |
| Column BD | −H·h/b | −13.333333 | −13.333333333333330 | ≈ 0 |
| Beam CD | −H | −20.000000 | −20.000000000000057 | $6 \times 10^{-14}$ |
| **Deflections [mm]** | | | | |
| Sway at C | eq. (3) | 3.348643 | 3.348642691912245 | ≈ 0 |
| Sway at D | $\delta_C - Hb/EA_c$ | 3.201302 | 3.201301841350706 | ≈ 0 |

| | | | | |
|---|---|---|---|---|
| Vertical at D | $N_{BD}\cdot h/EA_c$ | −0.065485 | −0.065484822471795 | ≈ 0 |
| Vertical at C | 0 | 0.000000 | 0.000000000000000 | 0 |

**4.2 LC2, nonlinear** (H to the left — BC active, AD slack)

| Quantity | Formula | Hand solution | FEM result | Difference |
|---|---|---|---|---|
| **Support reactions [kN]** | | | | |
| $A_x$ | 0 | 0.000000 | 0.000000000000000 | 0 |
| $A_z$ | $+H\cdot h/b$ | 13.333333 | 13.333333333333330 | ≈ 0 |
| $B_x$ | $+H$ | 20.000000 | 19.999999999999996 | ≈ 0 |
| $B_z$ | $-H\cdot h/b$ | −13.333333 | −13.333333333333330 | ≈ 0 |
| **Axial forces [kN] (tension +)** | | | | |
| Diagonal AD (slack) | 0 | 0.000000 | 0.000000000000000 | 0 |
| Diagonal BC (active) | $H\cdot L_d/b$ | 24.037009 | 24.037008503093261 | 0 |
| Column AC | $-H\cdot h/b$ | −13.333333 | −13.333333333333330 | ≈ 0 |
| Column BD | 0 | 0.000000 | 0.000000000000000 | 0 |
| Beam CD | 0 | 0.000000 | 0.000000000000000 | 0 |
| **Deflections [mm]** | | | | |
| Sway at C | eq. (4) | −3.201302 | −3.201301841350706 | ≈ 0 |
| Sway at D | = sway at C | −3.201302 | −3.201301841350706 | ≈ 0 |
| Vertical at C | $N_{AC}\cdot h/EA_c$ | −0.065485 | −0.065484822471795 | ≈ 0 |

**4.3 LC1, linear** (both diagonals acting)

| Quantity | Formula | Hand solution | FEM result | Difference |
|---|---|---|---|---|
| **Support reactions [kN]** | | | | |
| $A_x$ | $-N_{AD}\cdot b/L_d$ | −9.775050 | −9.775050231626702 | $1 \times 10^{-14}$ |
| $A_z$ | $-(N_{AC} + N_{AD}\cdot h/L_d)$ | −13.333333 | −13.333333333333321 | $1 \times 10^{-14}$ |
| $B_x$ | $N_{BC}\cdot b/L_d$ | −10.224950 | −10.224949768373280 | ≈ 0 |
| $B_z$ | $-(N_{BD} + N_{BC}\cdot h/L_d)$ | 13.333333 | 13.333333333333321 | $1 \times 10^{-14}$ |

| | | | | |
|---|---|---|---|---|
| **Axial forces [kN] (tension +)** | | | | |
| Diagonal AD | $N_0 + X \cdot n_1$ | 11.748148 | 11.748148276788742 | $1 \times 10^{-14}$ |
| Diagonal BC | $N_0 + X \cdot n_1$ | −12.288860 | −12.288860226304502 | ≈ 0 |
| Column AC | $N_0 + X \cdot n_1$ | 6.816633 | 6.816633178915519 | ≈ 0 |
| Column BD | $N_0 + X \cdot n_1$ | −6.516700 | −6.516700154417800 | ≈ 0 |
| Beam CD | $N_0 + X \cdot n_1$ | −9.775050 | −9.775050231626693 | $2 \times 10^{-14}$ |
| **Deflection [mm]** | | | | |
| Sway at C | $\Sigma\, N \cdot n_0 \cdot L/EA$ | 1.636658 | 1.636657526060593 | ≈ 0 |

**✓ All three states match to floating-point precision, including the exactly-zero force in the slack rod** VERIFIED

## 5. Combinations are not superposed

CO1 = 1.0·LC1 + 0.5·LC2 has a net load of +10 kN to the right. A nonlinear structure must be solved under that total load: the answer is LC1 at half the load (BC slack, AD at 12.0185 kN). Adding the two nonlinear case results instead would put tension in *both* rods — a state no load can produce. The solver solves the combination on its total load:

| Member force [kN] | Hand solution | FEM CO1 | Difference | LC1 + 0.5·LC2 (wrong) |
|---|---|---|---|---|
| Diagonal AD | 12.018504 | 12.018504252 | 0 | 24.037009 |
| Diagonal BC | 0.000000 | 0.000000000 | 0 | 12.018504 |
| Column AC | 0.000000 | 0.000000000 | 0 | −6.666667 |
| Column BD | −6.666667 | −6.666666667 | ≈ 0 | −13.333333 |
| Beam CD | −10.000000 | −10.000000000 | $3 \times 10^{-14}$ | −20.000000 |
| Sway at C [mm] | 1.674321 | 1.674321346 | ≈ 0 | 1.747992 |

**✓ The combination is solved on its total load, not superposed** VERIFIED

## 6. Final results

The design-relevant values of the bracing, by hand and by FEM:

| Quantity | Formula | Hand solution | FEM result | Difference |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| Max rod tension, LC1 (AD) [kN] | $H \cdot L_d / b$ | 24.037009 | 24.037008503093261 | 0 |
| Max rod tension, LC2 (BC) [kN] | $H \cdot L_d / b$ | 24.037009 | 24.037008503093261 | 0 |
| Max column compression, LC1 (BD) [kN] | $-H \cdot h / b$ | −13.333333 | −13.333333333333330 | ≈ 0 |
| Max uplift reaction, LC1 ($A_z$) [kN] | $-H \cdot h / b$ | −13.333333 | −13.333333333333330 | ≈ 0 |
| Sway at C, LC1 [mm] | eq. (3) | 3.348643 | 3.348642691912245 | ≈ 0 |
| Sway at C, LC2 [mm] | eq. (4) | −3.201302 | −3.201301841350706 | ≈ 0 |
| Rod tension, CO1 (AD) [kN] | $H_{net} \cdot L_d / b$ | 12.018504 | 12.018504251546631 | 0 |

**✓ Final: rod design force 24.0370 kN (either direction), column −13.3333 kN, uplift at the base 13.3333 kN, sway 3.3486 / 3.2013 mm — hand and FEM agree** VERIFIED

## 7. Conclusion

The FEM calculator's nonlinear solver finds the correct active rod for both load directions, drops the slack rod out to an exact zero, and reproduces the determinate method-of-joints forces, the support reactions and the virtual-work deflections to floating-point round-off. The linear solution with both rods acting matches the force method equally exactly, and a combination is solved on its total load rather than by adding nonlinear case results — the two things that make tension-only bracing different from an ordinary truss.

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The same comparison runs as part of the automated test suite on every test run: `tests/verificationExamples.test.ts` solves the project file `calc_tests/verification/x-braced-bay.json` and asserts its support reactions, deflections and internal forces against the hand solution above, recomputed from the sections and material stored in the file.