Verification — Simply Supported Beam under UDL

FEM Calculator core solver checked against the classical closed-form beam solution

Structure type	Simply supported beam (pin + roller), uniformly distributed load
Verification method	Euler-Bernoulli closed-form solution, derived by direct double integration
Quantities checked	Reactions, maximum shear, maximum bending moment, and maximum (midspan) deflection
Result	**Exact match** — FEM agrees with theory to $\approx 10^{-14}$ (floating-point round-off only)

1. Structure and loading

A single-span beam, $L = 6$ m, pinned at one end and on a roller at the other, carrying a uniformly distributed load $w = 10$ kN/m over its full length. This is the most fundamental bending benchmark in structural analysis — every quantity has an exact closed-form answer, so it isolates the beam-bending code path (shear/moment integration and the flexural stiffness matrix) with nothing else in play.

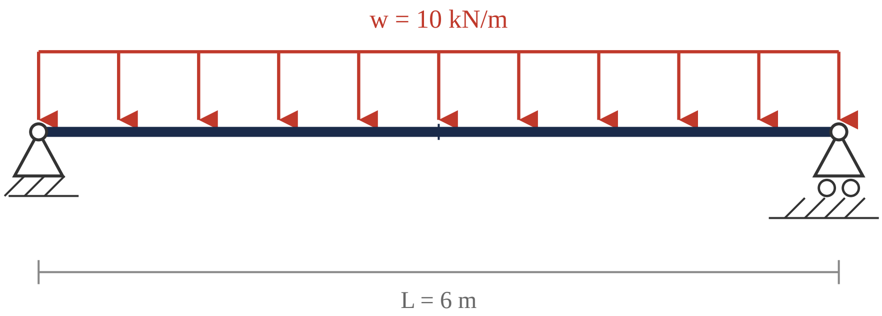

Fig. 1 — Simply supported beam under a full-span uniformly distributed load.

2. Hand solution (Euler-Bernoulli, direct integration)

Integrating $EI \cdot v'''' = w$ twice for shear and moment, then twice more for slope and deflection, with the boundary conditions $v(0) = v(L) = 0$:

> Reactions (symmetry): $R_A = R_B = wL/2 =$ **30 kN**
>
> Shear: $V(x) = wL/2 - wx \quad \rightarrow \quad V_{max} = wL/2 =$ **30 kN** (at each support)
>
> Moment: $M(x) = (w/2) \cdot x \cdot (L - x) \quad \rightarrow \quad M_{max} = wL^2/8 =$ **45 kN·m** (midspan)
>
> Deflection: $EI \cdot v(x) = (wL/12)x^3 - (w/24)x^4 - (wL^3/24)x$
>
> $\rightarrow \quad v_{max} = 5wL^4/(384EI)$ at midspan; end slope $|\theta| = wL^3/(24EI)$

3. FEM model

Built directly against the calculator's core solver (`analyze()`), bypassing the UI: one member (L=6 m, 200×400 mm rectangular section, S235-like steel, E=210 GPa), a pin at one end (translations restrained, bending rotation free), a roller at the other (same, but horizontal translation also free), and a full-span local distributed load in the vertical direction. Result stations are the solver's default 17-point grid along the member; the midspan value is read directly from that grid rather than interpolated.

4. Results comparison

Quantity	Formula	Hand solution	FEM result	Difference
Reaction R_A	wL/2	30.000000	29.999999999999993	7×10^{-15}
Reaction R_B	wL/2	30.000000	30.000000000000000	0
Max shear V_{max}	wL/2	30.000000	30.000000000000007	7×10^{-15}
Max moment M_{max}	$wL^2/8$	45.000000	44.999999999999970	3×10^{-14}
Midspan deflection [mm]	$5wL^4/384EI$	0.753348	0.753348214285714	$< 10^{-12}$
End deflections [mm]	0	0.000000	0.000000000000000	0

✓ All quantities match to floating-point precision VERIFIED

5. Conclusion

The FEM calculator reproduces the exact Euler-Bernoulli beam solution for a simply supported beam under a full-span UDL to within floating-point round-off, across reactions, shear, moment, and deflection simultaneously. Together with the earlier three-hinged-frame and king-post-truss checks, this covers the three main behaviors a general frame solver needs to get right: pure bending (this beam), combined axial + bending in a rigid frame, and pure axial (truss) action.

Generated as part of an ad-hoc verification exercise — not a permanent addition to the automated test suite. The equivalent scripted check (built and run against `src/core` directly via Vitest) was deleted after use, per this session's convention for one-off scratch verification.