

Verification — Timber Column, Buckling About Both Axes (EC5)

FEM Calculator timber assessment (EN 1995-1-1) checked against a hand calculation: flexural buckling about y and z with different buckling lengths, combined with bending

Structure type	Pinned glulam column GL24h, 140 × 200 mm, 4 m high; axial load from the roof plus wind along the height; braced about the weak axis at mid-height
Verification method	EN 1995-1-1 by hand: 6.3.2 relative slenderness (6.21/6.22), k_c (6.25–6.29) with $\beta_c = 0.1$, buckling interaction (6.23/6.24), cross-section checks 6.1.4, 6.1.6, 6.2.4 and 6.1.7; k_{mod} from the shortest action, k_h for glulam
Quantities checked	Design forces, k_{mod} , γ_M , k_h , design strengths, buckling lengths, λ_{rel} and k_c about both axes, and every utilization
Result	Match — internal forces to $\approx 10^{-15}$; everything the assessment reports to 3 decimals equals the hand value to that rounding. Governing: buckling about the strong axis, $\eta = 0.784$

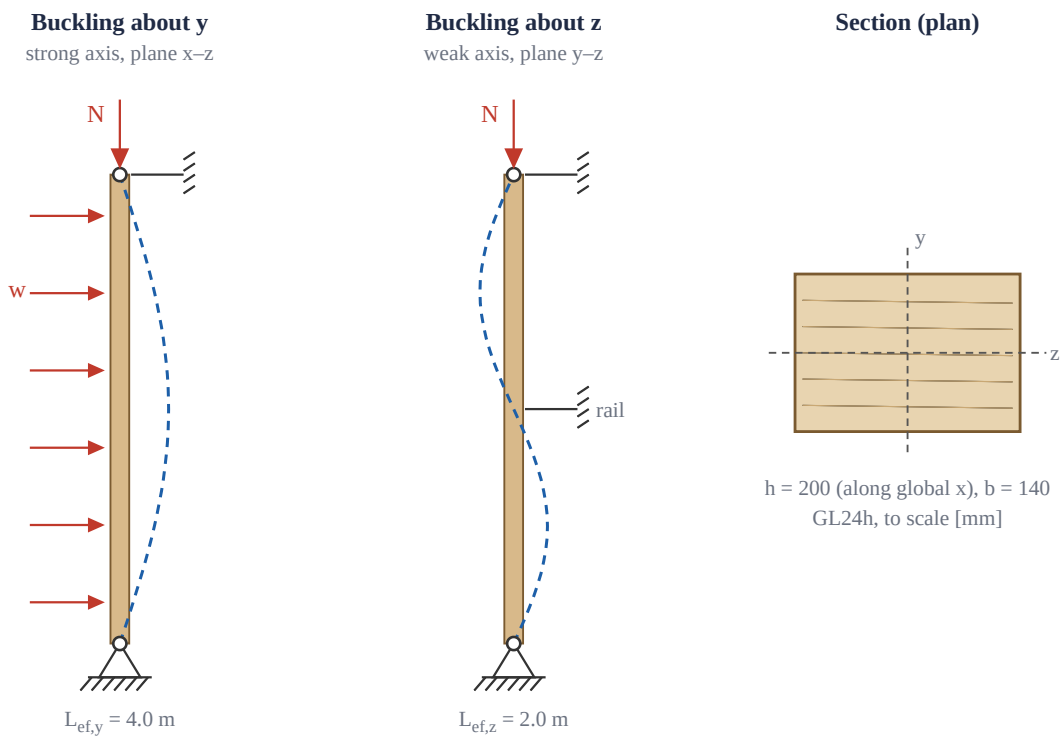


Fig. 1 — The column in its two buckling planes (buckled shapes dashed). About the strong axis y it is pinned top and bottom and buckles over its full height; about the weak axis z a wall rail holds it at mid-height, halving the buckling length.

1. Structure and loading

A glulam column GL24h (EN 14080: $f_{c,0,k} = f_{m,k} = 24$, $f_{v,k} = 3.5$, $E_{0,05} = 9\,600$ MPa), cross-section $b \times h = 140 \times 200$ mm, height $L = 4$ m, pinned at the base and held laterally at the top by the roof bracing. It carries the roof reaction at the top — permanent $G = 100$ kN and imposed $Q = 60$ kN — and a wind load $w = 1.0$ kN/m along its height, acting in the plane of the strong axis. Service class 1. The ULS combination is $CO1 = 1.35 \cdot G + 1.5 \cdot Q + 0.9 \cdot W$ (imposed leading, wind accompanying with $\psi_0 = 0.6$).

The column buckles differently in the two planes, which is the point of the example. About the **strong axis y** (bending in the plane of the wind) nothing restrains it between the ends: $L_{ef,y} = 1.0 \cdot L = 4.0$ m.

About the **weak axis z** a wall rail holds it at mid-height: $L_{ef,z} = 0.5 \cdot L = 2.0$ m. Neither axis is obviously critical in advance — the weak axis has the smaller radius of gyration, the strong axis the longer buckling length, and it also carries the bending.

2. Hand solution

Units: forces in N or kN, lengths in mm or m, stresses in MPa = N/mm².

2.1 Design forces.

$$N_d = 1.35 \cdot G + 1.5 \cdot Q = 1.35 \cdot 100 \text{ kN} + 1.5 \cdot 60 \text{ kN} = \mathbf{225.0 \text{ kN}}$$

$$M_{y,d} = 0.9 \cdot w \cdot L^2 / 8 = 0.9 \cdot 1.0 \text{ kN/m} \cdot (4 \text{ m})^2 / 8 = \mathbf{1.800 \text{ kNm}} \text{ (mid-height)}$$

$$V_{z,d} = 0.9 \cdot w \cdot L / 2 = 0.9 \cdot 1.0 \text{ kN/m} \cdot 4 \text{ m} / 2 = \mathbf{1.800 \text{ kN}} \text{ (ends)}$$

2.2 Design strengths. The combination contains permanent, medium-term (imposed) and short-term (wind) actions; 3.1.3(2) takes the shortest $\rightarrow$ short-term, service class 1 $\rightarrow k_{mod} = 0.9$ (Table 3.1); glulam $\gamma_M = 1.25$ (Table 2.3).

$$k_h = \min\{(600/h)^{0.1}; 1.1\} = \min\{(600 \text{ mm} / 200 \text{ mm})^{0.1}; 1.1\} = \min\{1.1161; 1.1\} = \mathbf{1.1} \quad (3.3(3))$$

$$f_{c,0,d} = k_{mod} \cdot f_{c,0,k} / \gamma_M = 0.9 \cdot 24 \text{ MPa} / 1.25 = \mathbf{17.2800 \text{ MPa}}$$

$$f_{m,d} = k_{mod} \cdot k_h \cdot f_{m,k} / \gamma_M = 0.9 \cdot 1.1 \cdot 24 \text{ MPa} / 1.25 = \mathbf{19.0080 \text{ MPa}}$$

$$f_{v,d} = k_{mod} \cdot f_{v,k} / \gamma_M = 0.9 \cdot 3.5 \text{ MPa} / 1.25 = \mathbf{2.5200 \text{ MPa}}$$

2.3 Stresses.

$$A = b \cdot h = 140 \text{ mm} \cdot 200 \text{ mm} = \mathbf{28\,000 \text{ mm}^2}, \quad W_y = b \cdot h^2 / 6 = 140 \text{ mm} \cdot (200 \text{ mm})^2 / 6 = \mathbf{933\,333 \text{ mm}^3}$$

$$\sigma_{c,0,d} = N_d / A = 225\,000 \text{ N} / 28\,000 \text{ mm}^2 = \mathbf{8.0357 \text{ MPa}}$$

$$\sigma_{m,y,d} = M_{y,d} / W_y = 1\,800\,000 \text{ Nmm} / 933\,333 \text{ mm}^3 = \mathbf{1.9286 \text{ MPa}}$$

2.4 Slenderness and k_c about both axes (6.21–6.22, 6.25–6.29; glulam $\beta_c = 0.1$; $\sqrt{(f_{c,0,k}/E_{0,05})} = \sqrt{(24 \text{ MPa} / 9\,600 \text{ MPa})} = 0.05000$):

y ($L_{ef,y} = 4\,000 \text{ mm}$): $i_y = h / \sqrt{12} = 200 \text{ mm} / \sqrt{12} = 57.735 \text{ mm}, \quad \lambda_y = L_{ef,y} / i_y = 4\,000 \text{ mm} / 57.735 \text{ mm} = 69.282$

$$\lambda_{rel,y} = (\lambda_y / \pi) \cdot \sqrt{(f_{c,0,k} / E_{0,05})} = (69.282 / \pi) \cdot 0.05000 = \mathbf{1.1027} \quad (6.21)$$

$$k_y = 0.5 \cdot [1 + \beta_c \cdot (\lambda_{rel,y} - 0.3) + \lambda_{rel,y}^2] = 0.5 \cdot [1 + 0.1 \cdot (1.1027 - 0.3) + 1.1027^2] = \mathbf{1.1481}$$

$$k_{c,y} = 1 / (k_y + \sqrt{(k_y^2 - \lambda_{rel,y}^2)}) = 1 / (1.1481 + \sqrt{(1.1481^2 - 1.1027^2)}) = \mathbf{0.6813} \quad (6.25)(6.27)$$

z ($L_{ef,z} = 2\,000 \text{ mm}$): $i_z = b / \sqrt{12} = 140 \text{ mm} / \sqrt{12} = 40.415 \text{ mm}, \quad \lambda_z = L_{ef,z} / i_z = 2\,000 \text{ mm} / 40.415 \text{ mm} = 49.487$

$$\lambda_{rel,z} = (49.487 / \pi) \cdot 0.05000 = \mathbf{0.7876} \quad (6.22)$$

$$k_z = 0.5 \cdot [1 + 0.1 \cdot (0.7876 - 0.3) + 0.7876^2] = \mathbf{0.8345} \quad (6.28)$$

$$k_{c,z} = 1 / (0.8345 + \sqrt{(0.8345^2 - 0.7876^2)}) = \mathbf{0.9005} \quad (6.26)$$

Both relative slendernesses exceed 0.3, so both buckling checks apply (6.3.2(3)). The strong axis is the more slender one here: the rail halves $L_{ef,z}$, more than compensating for i_z being 30 % smaller than i_y .

2.5 Checks (rectangular section, $k_m = 0.7$; $M_z = 0$):

Compression: $\sigma_{c,0,d} / f_{c,0,d} = 8.0357 \text{ MPa} / 17.2800 \text{ MPa} = \mathbf{0.4650} \quad (6.2)$

Bending: $\sigma_{m,y,d} / f_{m,d} = 1.9286 \text{ MPa} / 19.0080 \text{ MPa} = \mathbf{0.1015} \quad (6.11)$

Compression + bending: $(\sigma_{c,0,d} / f_{c,0,d})^2 + \sigma_{m,y,d} / f_{m,d} = (0.4650)^2 + 0.1015 = \mathbf{0.3177} \quad (6.19)$

Buckling about y: $\sigma_{c,0,d} / (k_{c,y} \cdot f_{c,0,d}) + \sigma_{m,y,d} / f_{m,d} = 8.0357 \text{ MPa} / (0.6813 \cdot 17.2800 \text{ MPa}) + 0.1015$

$$= 0.6825 + 0.1015 = \mathbf{0.7840} \quad (6.23)$$

Buckling about z: $\sigma_{c,0,d} / (k_{c,z} \cdot f_{c,0,d}) + k_m \cdot \sigma_{m,y,d} / f_{m,d} = 8.0357 \text{ MPa} / (0.9005 \cdot 17.2800 \text{ MPa}) + 0.7 \cdot 0.1015$

$$= 0.5164 + 0.0710 = \mathbf{0.5874} \quad (6.24)$$

Shear at the ends: $\tau_d = 1.5 \cdot V_{z,d} / (k_{cr} \cdot b \cdot h) = 1.5 \cdot 1\,800\text{ N} / (0.67 \cdot 28\,000\text{ mm}^2) = 0.1439\text{ MPa}$

$$\eta = \tau_d / f_{v,d} = 0.1439\text{ MPa} / 2.5200\text{ MPa} = \mathbf{0.0571} \quad (6.13)$$

Buckling about the strong axis governs with $\eta = 0.784$; the cross-section alone would be at 0.318.

Lateral-torsional buckling does not reduce the bending resistance ($L_{ef} = L = 4\,000\text{ mm}$, $G_{0,05} = 650\text{ MPa} \cdot 9\,600 / 11\,500 = 542.61\text{ MPa}$, $I_z = 200\text{ mm} \cdot (140\text{ mm})^3 / 12 = 45\,733\,333\text{ mm}^4$, $I_{tor} = 103\,873\,878\text{ mm}^4$ from the section):

$$\sigma_{m,crit} = \pi \cdot \sqrt{(E_{0,05} \cdot I_z \cdot G_{0,05} \cdot I_{tor}) / (L_{ef} \cdot W_y)} = \pi \cdot \sqrt{(9\,600\text{ MPa} \cdot 45\,733\,333\text{ mm}^4 \cdot 542.61\text{ MPa} \cdot 103\,873\,878\text{ mm}^4) / (4\,000\text{ mm} \cdot 933\,333\text{ mm}^3)} = 132.37\text{ MPa} \quad (6.31)$$

$$\lambda_{rel,m} = \sqrt{(24\text{ MPa} / 132.37\text{ MPa})} = 0.4258 \leq 0.75 \rightarrow k_{crit} = 1 \quad (6.34)$$

So no separate LTB check is needed, and the assessment correctly reports none.

3. FEM model and assessment settings

The structure is stored as a regular project file, `timber-column-buckling.json`: one vertical member, base node 1 pinned (translations held), top node 2 held in x and y but free to move vertically. Three load cases — G (permanent, -100 kN at the top), Q (imposed, -60 kN) and W (wind, 1 kN/m in global x along the member) — and $CO1 = 1.35 \cdot G + 1.5 \cdot Q + 0.9 \cdot W$. The member carries its own buckling lengths $L_{ef,y} = 4.0\text{ m}$ and $L_{ef,z} = 2.0\text{ m}$ (the mid-height rail is represented by the shorter z-length, not modelled as a support). The forces come from the core solver (`analyze()`), the checks from the timber assessment (`assessTimber()` , *Assessment* → *Timber*) on $CO1$ with its default settings: service class 1, load duration derived from the actions, glulam $\gamma_M = 1.25$, k_h on, $\beta_c = 0.1$, and the member's own buckling lengths used.

4. Results comparison

Quantity	Formula	Hand solution	FEM / assessment	Difference
Design internal forces (FEM, CO1)				
N_d [kN] (compression)	$1.35 \cdot G + 1.5 \cdot Q$	225.0000	225.00000000000000	0
$M_{y,d}$ at mid-height [kNm]	$0.9 \cdot w \cdot L^2 / 8$	1.8000	1.8000000000000000	0
$V_{z,d}$ at the ends [kN]	$0.9 \cdot w \cdot L / 2$	1.8000	1.8000000000000000	0
Design parameters (assessment, reported to 3 decimals)				

k_{mod} (wind is the shortest action)	Table 3.1	0.900	0.900	0
γ_M (glulam)	Table 2.3	1.250	1.250	0
k_h for M_y (capped)	$\min\{(600/h)^{0.1}; 1.1\}$	1.100	1.100	0
$f_{c,0,d}$ [MPa]	$k_{\text{mod}} \cdot f_{c,0,k} / \gamma_M$	17.2800	17.280	0
$f_{m,d}$ [MPa]	$k_{\text{mod}} \cdot k_h \cdot f_{m,k} / \gamma_M$	19.0080	19.008	≈ 0
$L_{\text{ef},y} / L_{\text{ef},z}$ [m]	member	4.000 / 2.000	4.000 / 2.000	0
Buckling about y — strong axis (assessment working, 3 decimals)				
$\lambda_{\text{rel},y}$	(6.21)	1.1027	1.103	3×10^{-4} (rounding)
$k_{c,y}$	(6.25)	0.6813	0.681	3×10^{-4} (rounding)
Buckling about z — weak axis (assessment working, 3 decimals)				
$\lambda_{\text{rel},z}$	(6.22)	0.7876	0.788	4×10^{-4} (rounding)
$k_{c,z}$	(6.26)	0.9005	0.901	5×10^{-4} (rounding)
Utilizations at mid-height (assessment, 3 decimals)				
Compression, 6.2	$\sigma_{c,0,d} / f_{c,0,d}$	0.4650	0.465	3×10^{-5} (rounding)
Bending, 6.11	$\sigma_{m,y,d} / f_{m,d}$	0.1015	0.101	5×10^{-4} (rounding)
Compression + bending, 6.19	$(\sigma_c / f_c)^2 + \sigma_m / f_m$	0.3177	0.318	3×10^{-4} (rounding)
Buckling about y, 6.23	$\sigma_c / (k_{c,y} f_c) + \sigma_m / f_m$	0.7840	0.784	2×10^{-6} (rounding)
Buckling about z, 6.24	$\sigma_c / (k_{c,z} f_c) + k_m \sigma_m / f_m$	0.5874	0.587	4×10^{-4} (rounding)
Shear at the ends, 6.13	$\tau_d / f_{v,d}$	0.0571	0.057	1×10^{-4} (rounding)
Governing (overall)	max	0.7840	0.784	2×10^{-6} (rounding)

The assessment reports design strengths, slenderness, k_c and utilizations rounded to 3 decimals; those rows agree to the last reported digit, and the differences shown are that rounding ($\leq 5 \times 10^{-4}$), not a calculation error. The internal forces are unrounded.

✓ All quantities match; governing check buckling about y, $\eta = 0.784$ **VERIFIED**

5. Final design value

The governing check is buckling about the strong axis (6.23). Both of its terms are linear in the loads, so with G, Q and W raised together by the factor $\alpha = 1/\eta$ the column reaches $\eta = 1$ exactly; the other checks stay below 1 at that load ((6.19): $(0.4650 \cdot 1.2755)^2 + 0.1015 \cdot 1.2755 = 0.4812$; (6.24): $0.5874 \cdot 1.2755 = 0.7493$). The final design values:

$$\eta_{\max} = \mathbf{0.7840} \text{ (6.23)} \rightarrow \alpha = 1/\eta_{\max} = 1 / 0.7840 = \mathbf{1.2755}$$

$$N_{d,\max} = \alpha \cdot N_d = 1.2755 \cdot 225.0 \text{ kN} = \mathbf{286.99 \text{ kN}} \quad \text{with } M_{y,d} = \alpha \cdot M_{y,d} = 1.2755 \cdot 1.800 \text{ kNm} = 2.296 \text{ kNm}$$

$$\text{For axial load alone (no wind): } N_{b,y,Rd} = k_{c,y} \cdot f_{c,0,d} \cdot A = 0.6813 \cdot 17.2800 \text{ MPa} \cdot 28\,000 \text{ mm}^2 = \mathbf{329.65 \text{ kN}}$$

The assessment gives the utilization; the FEM value of $N_{d,\max}$ follows from it as N_d/η , with the FEM axial force:

Final value	Formula	Hand solution	FEM / assessment	Difference
η , buckling about y (6.23)	$\sigma_c/(k_{c,y}f_c) + \sigma_m/f_m$	0.7840	0.784	2×10^{-6} (rounding)
$N_{d,\max}$ [kN] (proportional)	N_d/η	286.990	286.990	7×10^{-4} (rounding)
$N_{b,y,Rd}$ [kN] (axial only)	$k_{c,y} \cdot f_{c,0,d} \cdot A$	329.652	329.495	2×10^{-1} (rounding)

The assessment reports η and k_c to 3 decimals, so the values derived from them agree with the hand values to that rounding.

✓ Final design: $\eta = 0.784$ (buckling about y), $N_{d,\max} = 287.0 \text{ kN}$ under proportional loading, $N_{b,y,Rd} = 329.7 \text{ kN}$ for axial load alone — hand and FEM agree **VERIFIED**

6. Conclusion

The FEM calculator's timber assessment reproduces the EN 1995-1-1 hand calculation of a glulam column buckling about two axes with different buckling lengths: k_{mod} from the shortest-duration action in the combination, the glulam partial factor and the capped size factor, the relative slenderness and k_c

about each axis with the glulam β_c , both buckling interactions (6.23/6.24) with k_m on the correct side, and the cross-section checks. It identifies strong-axis buckling as governing, as the hand calculation does, and correctly omits lateral-torsional buckling for this stocky section. The final design value $N_{d,max} = 287.0$ kN follows from its utilization exactly as from the hand calculation.

The same comparison runs as part of the automated test suite on every test run:

`tests/verificationExamples.test.ts` solves the project file `calc_tests/verification/timber-column-buckling.json`, runs the timber assessment on CO1 and asserts the internal forces, design parameters, slenderness, k_c , utilizations and the final design value against the hand calculation above, recomputed from the material and section stored in the file.