

Verification — Timber Beam in Bending (EC5)

FEM Calculator timber assessment (EN 1995-1-1) checked against a hand calculation: bending about the strong axis, shear and lateral-torsional buckling

Structure type	Simply supported solid timber beam C24, 80 × 240 mm, span 5 m, uniform imposed load, no lateral restraint between the supports
Verification method	EN 1995-1-1 by hand: 6.1.6 bending (6.11), 6.1.7 shear (6.13) with k_{cr} , 6.3.3 lateral-torsional buckling (6.31, 6.33, 6.34); k_{mod} from Table 3.1 and the load-duration class of the action
Quantities checked	Design internal forces, k_{mod} , k_h , design strengths, effective length, and the utilization of every check
Result	Match — internal forces to $\approx 10^{-15}$; strengths and utilizations equal to the 3 decimals the assessment reports. Governing: lateral-torsional buckling, $\eta = 0.922$

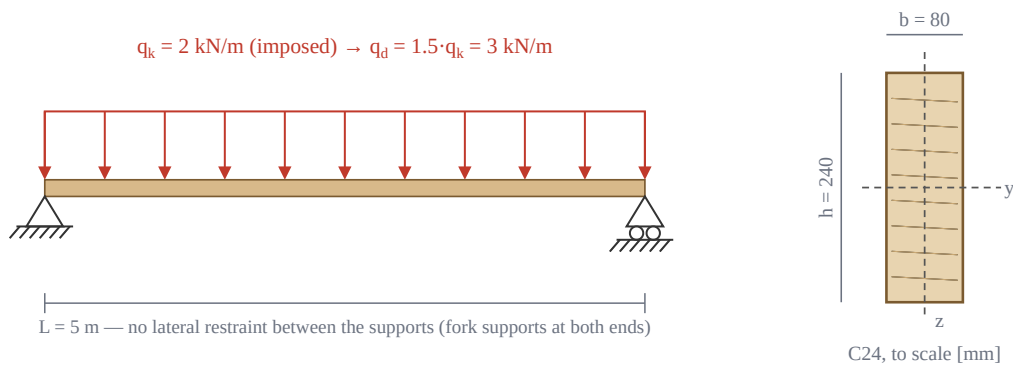


Fig. 1 — Elevation and cross-section. The load acts in the plane of the strong axis, so the beam bends about y only (M_y); with no lateral restraint it can buckle sideways.

1. Structure and loading

A floor beam of solid softwood C24 (EN 338: $f_{m,k} = 24$, $f_{v,k} = 4.0$, $E_{0,mean} = 11\,000$, $E_{0,05} = 7\,400$, $G_{mean} = 690\text{ MPa}$), cross-section $b \times h = 80 \times 240\text{ mm}$, spans $L = 5\text{ m}$ between a pin and a roller. It carries an imposed load $q_k = 2\text{ kN/m}$ (self-weight is left out to keep the numbers round). Service class 1 (heated interior). The ULS combination is CO1 = 1.5 · Q, so $q_d = 3\text{ kN/m}$. Both supports are fork supports (twisting prevented, as the model's r_x restraint gives), but the compression edge is free along the span — a beam that can buckle laterally, which is what makes it a useful check.

2. Hand solution

Units: forces in N or kN, lengths in mm or m, stresses in MPa = N/mm².

2.1 Design forces.

$$q_d = \gamma_Q \cdot q_k = 1.5 \cdot 2 \text{ kN/m} = \mathbf{3.000 \text{ kN/m}}$$

$$M_{y,d} = q_d \cdot L^2 / 8 = 3 \text{ kN/m} \cdot (5 \text{ m})^2 / 8 = \mathbf{9.375 \text{ kNm}} \text{ (midspan)}$$

$$V_{z,d} = q_d \cdot L / 2 = 3 \text{ kN/m} \cdot 5 \text{ m} / 2 = \mathbf{7.500 \text{ kN}} \text{ (support)}$$

2.2 Design strengths. The only action is imposed load, which Table 2.2 classes as *medium-term*; with service class 1, Table 3.1 gives $k_{\text{mod}} = 0.8$. Solid timber: $\gamma_M = 1.3$. The size factor k_h (3.2(3)) only raises strengths for depths below 150 mm; here $h = 240 \text{ mm}$, so $k_h = 1$.

$$f_{m,d} = k_{\text{mod}} \cdot k_h \cdot f_{m,k} / \gamma_M = 0.8 \cdot 1.0 \cdot 24 \text{ MPa} / 1.3 = \mathbf{14.7692 \text{ MPa}}$$

$$f_{v,d} = k_{\text{mod}} \cdot f_{v,k} / \gamma_M = 0.8 \cdot 4.0 \text{ MPa} / 1.3 = \mathbf{2.4615 \text{ MPa}}$$

2.3 Bending, 6.1.6. With $M_z = 0$, both (6.11) and (6.12) reduce to one stress ratio, and (6.11) governs:

$$W_y = b \cdot h^2 / 6 = 80 \text{ mm} \cdot (240 \text{ mm})^2 / 6 = \mathbf{768\,000 \text{ mm}^3}$$

$$\sigma_{m,y,d} = M_{y,d} / W_y = 9\,375\,000 \text{ Nmm} / 768\,000 \text{ mm}^3 = \mathbf{12.2070 \text{ MPa}}$$

$$\eta = \sigma_{m,y,d} / f_{m,d} = 12.2070 \text{ MPa} / 14.7692 \text{ MPa} = \mathbf{0.8265} \quad (6.11)$$

2.4 Shear, 6.1.7. The crack factor $k_{\text{cr}} = 0.67$ (recommended value) reduces the effective width:

$$\tau_d = 1.5 \cdot V_{z,d} / (k_{\text{cr}} \cdot b \cdot h) = 1.5 \cdot 7\,500 \text{ N} / (0.67 \cdot 80 \text{ mm} \cdot 240 \text{ mm}) = \mathbf{0.8745 \text{ MPa}}$$

$$\eta = \tau_d / f_{v,d} = 0.8745 \text{ MPa} / 2.4615 \text{ MPa} = \mathbf{0.3553} \quad (6.13)$$

2.5 Lateral-torsional buckling, 6.3.3. For a simply supported beam under a uniform load, Table 6.1 gives $L_{\text{ef}} = 0.9 \cdot L$ (load on the centroid axis). The critical bending stress uses the general expression (6.31); $G_{0,05}$ is taken in the same ratio to G_{mean} as $E_{0,05}$ to $E_{0,\text{mean}}$, and I_{tor} is the rectangle's torsion constant:

$$L_{\text{ef}} = 0.9 \cdot L = 0.9 \cdot 5\,000 \text{ mm} = \mathbf{4\,500 \text{ mm}} \quad (\text{Table 6.1})$$

$$G_{0,05} = G_{\text{mean}} \cdot E_{0,05} / E_{0,\text{mean}} = 690 \text{ MPa} \cdot 7\,400 \text{ MPa} / 11\,000 \text{ MPa} = \mathbf{464.18 \text{ MPa}}$$

$$I_z = h \cdot b^3 / 12 = 240 \text{ mm} \cdot (80 \text{ mm})^3 / 12 = \mathbf{10\,240\,000 \text{ mm}^4}$$

$$I_{\text{tor}} = h \cdot b^3 \cdot [1/3 - 0.21 \cdot (b/h) \cdot (1 - b^4/(12 \cdot h^4))] = 240 \text{ mm} \cdot (80 \text{ mm})^3 \cdot [1/3 - 0.21 \cdot (80/240) \cdot (1 - 80^4/(12 \cdot 240^4))]$$

$$= 122\,880\,000 \text{ mm}^4 \cdot 0.26341 = \mathbf{32\,367\,249 \text{ mm}^4}$$

$$\sigma_{\text{m,crit}} = \pi \cdot \sqrt{(E_{0,05} \cdot I_z \cdot G_{0,05} \cdot I_{\text{tor}}) / (L_{\text{ef}} \cdot W_y)}$$

$$= \pi \cdot \sqrt{(7\,400 \text{ MPa} \cdot 10\,240\,000 \text{ mm}^4 \cdot 464.18 \text{ MPa} \cdot 32\,367\,249 \text{ mm}^4) / (4\,500 \text{ mm} \cdot 768\,000 \text{ mm}^3)} = \mathbf{30.6718 \text{ MPa}} \quad (6.31)$$

$$\lambda_{\text{rel,m}} = \sqrt{(f_{\text{m,k}} / \sigma_{\text{m,crit}})} = \sqrt{(24 \text{ MPa} / 30.6718 \text{ MPa})} = \mathbf{0.8846} \quad (6.30)$$

$$0.75 < \lambda_{\text{rel,m}} \leq 1.4: \quad k_{\text{crit}} = 1.56 - 0.75 \cdot \lambda_{\text{rel,m}} = 1.56 - 0.75 \cdot 0.8846 = \mathbf{0.8966} \quad (6.34)$$

$$\eta = \sigma_{\text{m,y,d}} / (k_{\text{crit}} \cdot f_{\text{m,d}}) = 12.2070 \text{ MPa} / (0.8966 \cdot 14.7692 \text{ MPa}) = \mathbf{0.9219} \quad (6.33)$$

Lateral-torsional buckling governs: the beam would pass the cross-section check with $\eta = 0.827$, but its free compression edge costs about 10 % of the bending resistance. For comparison, the softwood shortcut (6.32) assumes the thin-strip torsion constant $h \cdot b^3/3$ and so overrates a stocky section:

$$\sigma_{\text{m,crit}} = 0.78 \cdot b^2 \cdot E_{0,05} / (h \cdot L_{\text{ef}}) = 0.78 \cdot (80 \text{ mm})^2 \cdot 7\,400 \text{ MPa} / (240 \text{ mm} \cdot 4\,500 \text{ mm}) = 34.20$$

$$\text{MPa} \quad (6.32)$$

$$\rightarrow \lambda_{\text{rel,m}} = \sqrt{(24 \text{ MPa} / 34.20 \text{ MPa})} = 0.8377 \rightarrow k_{\text{crit}} = 1.56 - 0.75 \cdot 0.8377 = 0.932$$

The assessment uses (6.31), the safe side.

3. FEM model and assessment settings

The structure is stored as a regular project file, `timber-beam-bending.json`: one member, pin at node 1 and roller at node 2 (both restrained against twisting, r_x), one load case *LC1 Imposed* (action: imposed, $q = -2 \text{ kN/m}$ in Z) and the combination $\text{CO1} = 1.5 \cdot \text{LC1}$. The member carries its own lateral-torsional buckling length $L_{\text{ef}} = 4.5 \text{ m}$. The design forces come from the core solver (`analyze()`) and the checks from the timber assessment (`assessTimber()`, *Assessment* $\rightarrow$ *Timber* in the app) on CO1, with its default settings: service class 1, load duration derived from the actions (CO1 $\rightarrow$ imposed $\rightarrow$ medium-term), $\gamma_M = 1.3$, k_h on, $k_{\text{cr}} = 0.67$, $k_{\text{sys}} = 1$, and the member's own buckling lengths used. Opening the file and running the timber assessment on CO1 reproduces every number below.

4. Results comparison

Quantity	Formula	Hand solution	FEM / assessment	Difference
Design internal forces (FEM, CO1 = 1.5·Q)				
$M_{y,d}$ at midspan [kNm]	$q_d L^2/8$	9.3750	9.374999999999995	≈ 0
$V_{z,d}$ at support [kN]	$q_d L/2$	7.5000	7.500000000000003	≈ 0
Design parameters (assessment, reported to 3 decimals)				
k_{mod} (from the load case action)	Table 3.1	0.800	0.800	0
k_h for M_y	3.2(3), $h \geq 150$	1.000	1.000	0
$f_{m,d}$ [MPa]	$k_{mod} \cdot k_h \cdot f_{m,k}/\gamma_M$	14.7692	14.769	2×10^{-4} (rounding)
$f_{v,d}$ [MPa]	$k_{mod} \cdot f_{v,k}/\gamma_M$	2.4615	2.462	5×10^{-4} (rounding)
L_{ef} for LTB [m]	$0.9 \cdot L$ (member)	4.500	4.500	0
Utilizations (assessment, reported to 3 decimals)				
Bending, 6.11	$\sigma_{m,y,d}/f_{m,d}$	0.8265	0.827	5×10^{-4} (rounding)
Shear, 6.13	$\tau_d/f_{v,d}$	0.3553	0.355	3×10^{-4} (rounding)
Lateral-torsional buckling, 6.33	$\sigma_{m,y,d}/(k_{crit} \cdot f_{m,d})$	0.9219	0.922	1×10^{-4} (rounding)
Governing (overall)	max	0.9219	0.922	1×10^{-4} (rounding)

The assessment reports design strengths and utilizations rounded to 3 decimals, so those rows agree to the last reported digit; the differences shown are that rounding ($\leq 5 \times 10^{-4}$), not a calculation error. The internal forces are unrounded and agree to round-off.

✓ All quantities match; governing check lateral-torsional buckling, $\eta = 0.922$ **VERIFIED**

5. Final design value

The governing check is lateral-torsional buckling (6.33). Its utilization, the design resistance moment it implies and the largest design load the beam can carry:

$$M_{b,Rd} = k_{crit} \cdot f_{m,d} \cdot W_y = 0.8966 \cdot 14.7692 \text{ MPa} \cdot 768\,000 \text{ mm}^3 = \mathbf{10.1695 \text{ kNm}}$$

$$q_{d,max} = 8 \cdot M_{b,Rd} / L^2 = 8 \cdot 10.1695 \text{ kNm} / (5 \text{ m})^2 = \mathbf{3.2543 \text{ kN/m}} \rightarrow q_{k,max} = q_{d,max} / \gamma_Q = 3.2543 \text{ kN/m} / 1.5 = \mathbf{2.1695 \text{ kN/m}}$$

The assessment gives the utilization; the FEM values of $M_{b,Rd}$ and $q_{d,max}$ follow from it as $M_{y,d}/\eta$ and q_d/η , with the FEM design moment $M_{y,d}$:

Final value	Formula	Hand solution	FEM / assessment	Difference
η , lateral-torsional buckling (6.33)	$\sigma_{m,y,d} / (k_{crit} \cdot f_{m,d})$	0.9219	0.922	1×10^{-4} (rounding)
$M_{b,Rd}$ [kNm]	$k_{crit} \cdot f_{m,d} \cdot W_y \mid M_{y,d} / \eta$	10.1695	10.1681	1×10^{-3} (rounding)
$q_{d,max}$ [kN/m]	$8 \cdot M_{b,Rd} / L^2 \mid q_d / \eta$	3.2543	3.2538	5×10^{-4} (rounding)

The assessment reports η to 3 decimals (0.922), so $M_{b,Rd}$ and $q_{d,max}$ derived from it agree with the hand values to that rounding (relative difference $\leq 5 \times 10^{-4} / \eta$).

✓ **Final design: $\eta = 0.922$ (lateral-torsional buckling), $M_{b,Rd} = 10.17 \text{ kNm}$, $q_{d,max} = 3.25 \text{ kN/m}$ — hand and FEM agree** VERIFIED

6. Conclusion

The FEM calculator's timber assessment reproduces the EN 1995-1-1 hand calculation of a laterally unrestrained C24 beam in strong-axis bending: k_{mod} derived from the action's load-duration class, the size factor correctly left at 1 for a 240 mm deep section, the design strengths, the bending (6.11) and shear (6.13) checks with the crack factor, and the lateral-torsional buckling check with $\sigma_{m,crit}$ from the general expression (6.31). It identifies lateral-torsional buckling as the governing check, as the hand calculation does, and the design resistance $M_{b,Rd} = 10.17 \text{ kNm}$ that follows from it.

The same comparison runs as part of the automated test suite on every test run:

`tests/verificationExamples.test.ts` solves the project file `calc_tests/verification/timber-beam-bending.json`, runs the timber assessment on CO1 and asserts the internal forces, design parameters, utilizations and the final design value against the hand calculation above, recomputed from the material and section stored in the file.