Verification — Three-Hinged Portal Frame

FEM Calculator core solver checked against an independently derived hand (statics) solution

Structure type	Statically determinate three-hinged portal frame (pinned bases + midspan beam hinge)
Verification method	Closed-form equilibrium solution (derived from first principles, not a memorized textbook value)
Quantities checked	Support reactions (Ax, Ay, Dx, Dy) and bending moment at the rigid column/beam joint and at the hinge
Result	**Exact match** — FEM agrees with hand statics to $\approx 10^{-13}$ (floating-point round-off only)

1. Structure and loading

A rectangular portal frame, 6 m span × 3 m tall, pin-supported at both bases (A and D). An internal hinge is placed at E, the midspan of the beam BC — this is the feature that makes the frame statically **determinate**: 4 unknown reactions (A_x, A_y, D_x, D_y) exactly match 4 available equations (3 global equilibrium equations + 1 "zero moment at the hinge" condition). A horizontal point load H = 10 kN is applied at joint B, at the top of the left column.

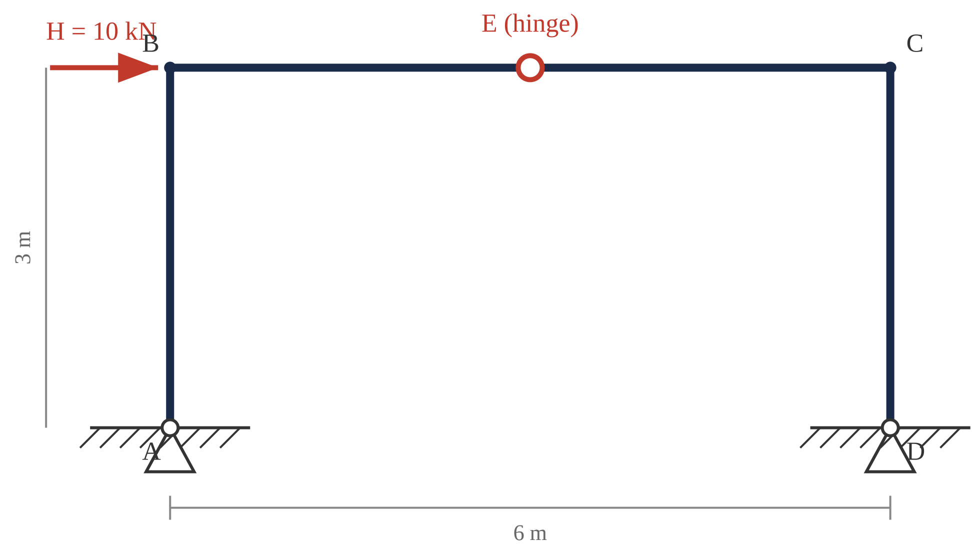

Fig. 1 — Frame geometry, pin supports (A, D), internal hinge (E), and applied load.

2. Hand solution (statics)

Taking moments about base A (counter-clockwise positive), then applying global equilibrium and the zero-moment condition at the hinge (free body A–B–E):

$$\Sigma M_A = 0: \quad 6 \cdot D_y - 3 \cdot H = 0 \quad \rightarrow \quad D_y = H/2 = \mathbf{5\ kN}$$

$$\Sigma F_y = 0: \quad A_y + D_y = 0 \quad \rightarrow \quad A_y = \mathbf{-5\ kN}$$

> Hinge ($\Sigma M_E = 0$ on A–B–E; H acts at the same height as E ⇒ zero arm): $-3 \cdot A_y + 3 \cdot A_x = 0 \rightarrow A_x = A_y =$ **−5 kN**
>
> $\Sigma F_x = 0$: $A_x + D_x + H = 0 \rightarrow D_x =$ **−5 kN**
>
> Column moment at B: $M(z) = z \cdot A_x \rightarrow |M_B| = 3 \times 5 =$ **15 kN·m**

Because the frame is statically determinate, this result depends only on equilibrium — it is independent of member stiffness (E, I, A), which is itself a useful property to check for.

3. FEM model

The identical geometry was built directly against the calculator's core solver (`analyze()`), bypassing the UI: pinned supports at A and D (translations restrained, rotations free), the beam split into two members at E with an in-plane bending release (`my`) on one side to model the hinge, and out-of-plane bracing at every free joint to keep the frame planar. Section/material values were arbitrary (S235-like steel, 200×300 mm rectangular section) — consistent with the determinacy argument above, the exact values used cannot affect the result.

4. Results comparison

Quantity	Hand solution	FEM result	Difference
Reaction A_x	−5.000000	−4.999999999999767	2.3×10^{-13}
Reaction A_y	−5.000000	−4.999999999999769	2.3×10^{-13}
Reaction D_x	−5.000000	−4.999999999999769	2.3×10^{-13}
Reaction D_y	+5.000000	4.999999999999771	2.3×10^{-13}
Moment at B (column side)	15.000000	14.999999999999297	7.0×10^{-13}
Moment at B (beam side)	15.000000	14.999999999999310	6.9×10^{-13}
Moment at hinge E	0.000000	3.55×10^{-15}	≈ 0

✓ All quantities match to floating-point precision VERIFIED

5. Conclusion

The FEM calculator reproduces the exact statically-determinate hand solution for this three-hinged portal frame to within floating-point round-off ($\sim 10^{-13}$), for both the support reactions and the internal bending moments. This also confirms two mechanisms work correctly together: pinned-support modeling and member end-release (hinge) modeling.

Generated as part of an ad-hoc verification exercise — not a permanent addition to the automated test suite. The equivalent scripted check (built and run against `src/core` directly via Vitest) was deleted after use, per this session's convention for one-off scratch verification.