

Verification — Steel Column Buckling, Class 3 and Class 4 (EC3)

FEM Calculator steel assessment (EN 1993-1-1, EN 1993-1-5) checked against a hand calculation: flexural buckling of two hollow-section columns, one class 3 and one class 4, with the effective area of the class 4 section

Structure type	Two pinned columns, $L = 10$ m, S355, square hollow sections EN 10219 with the same 10 mm wall: MSH 350×350×10 (class 3) and MSH 400×400×10 (class 4); axial load only
Verification method	EN 1993-1-1 by hand: classification Table 5.2, compression 6.2.4, flexural buckling 6.3.1 (curve c); effective width of the class 4 walls by EN 1993-1-5 §4.4 (internal element, $\psi = 1$, $k_\sigma = 4$) and $A_{\text{eff}} = A - \Delta A$
Quantities checked	Support reactions, axial force, axial shortening; class; ρ and A_{eff} ; N_{Rk} , λ , Φ , χ , $N_{b,Rd}$; every utilization
Result	Exact match — forces, shortening and utilizations to $\approx 10^{-15}$; the working the report prints (2–3 decimals) equals the hand value to that rounding. Governing: column 1, $\eta = 0.839$

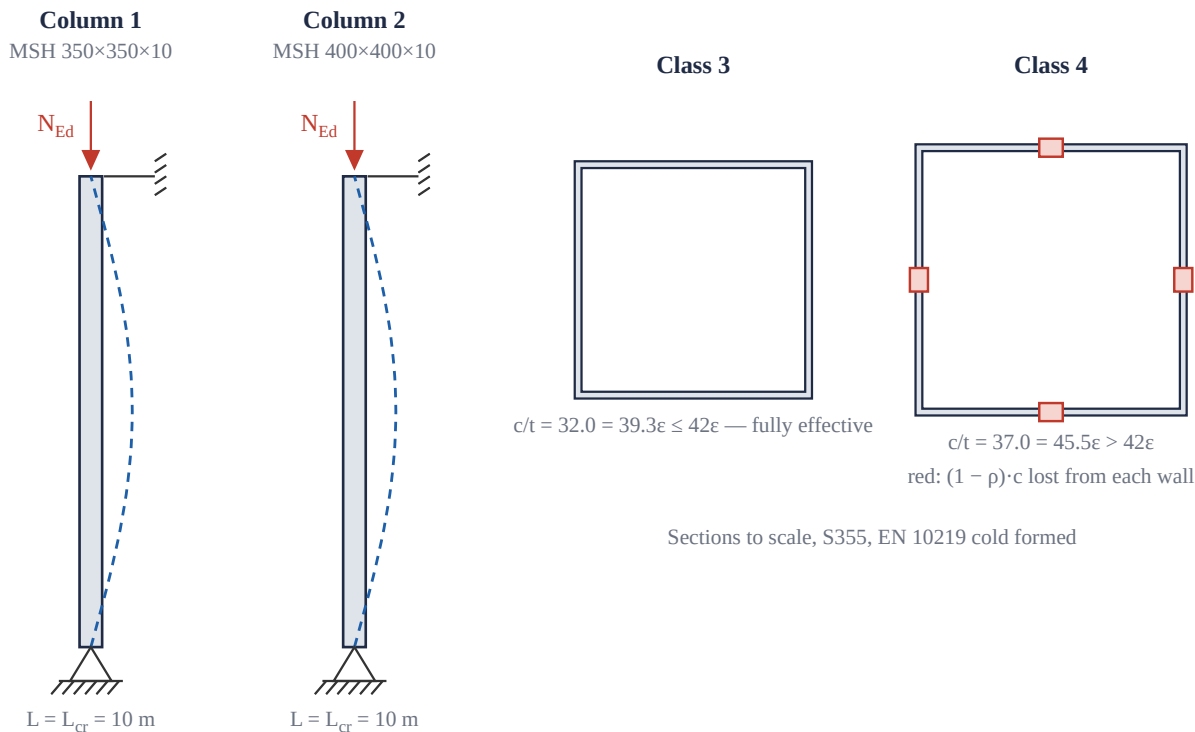


Fig. 1 — Left: the two columns, pinned at both ends, with their buckling shape (dashed). Right: the two sections to the same scale. Every wall of the class 3 section is fully effective; each wall of the class 4 section loses a strip $(1 - \rho) \cdot c$ at its middle, where it buckles locally.

1. Structure and loading

Two hall columns of height $L = 10$ m, pinned at the base and held laterally at the top in both directions, so $L_{cr,y} = L_{cr,z} = 10$ m. Both are cold-formed square hollow sections to EN 10219 in S355 ($f_y = 355$ MPa for $t \leq 40$ mm, $E = 210$ GPa, $\epsilon = \sqrt{(235/355)} = 0.8136$) with the same wall, $t = 10$ mm. Column 1 is MSH 350×350×10 ($A = 132.57$ cm², $i = 137.84$ mm), column 2 is MSH 400×400×10 ($A = 152.57$ cm², $i = 158.27$ mm). Each carries a permanent load $G = 1000$ kN and an imposed load $Q = 600$ kN at the top; self-weight is not included. $CO1 = 1.35 \cdot G + 1.5 \cdot Q$.

The two sizes were chosen to sit on either side of the class 3/4 boundary in compression. The larger section is stiffer against overall buckling, but its walls are too slender to reach f_y before they buckle locally, so part of each wall is discounted. The example checks both effects and how they combine.

2. Hand solution

Units: forces in N or kN, lengths in mm, stresses in MPa = N/mm².

2.1 Design force and shortening.

$$N_{Ed} = 1.35 \cdot G + 1.5 \cdot Q = 1.35 \cdot 1000 \text{ kN} + 1.5 \cdot 600 \text{ kN} = \mathbf{2250 \text{ kN}}$$
 in each column

Axial shortening under G alone, $\Delta = G \cdot L / (E \cdot A)$: column 1: $1\,000\,000 \text{ N} \cdot 10\,000 \text{ mm} / (210\,000 \text{ MPa} \cdot 13256.6 \text{ mm}^2) = \mathbf{3.5921 \text{ mm}}$; column 2: $1\,000\,000 \text{ N} \cdot 10\,000 \text{ mm} / (210\,000 \text{ MPa} \cdot 15256.6 \text{ mm}^2) = \mathbf{3.1212 \text{ mm}}$

2.2 Classification (Table 5.2, internal part in compression; for a hollow section $c = b - 3t$; $\epsilon = \sqrt{(235/f_y)}$ = $\sqrt{(235 \text{ MPa} / 355 \text{ MPa})} = 0.8136$):

$$\text{Limits: } 33\epsilon = 33 \cdot 0.8136 = 26.85, \quad 38\epsilon = 38 \cdot 0.8136 = 30.92, \quad 42\epsilon = 42 \cdot 0.8136 = 34.17$$

$$\text{Column 1: } c/t = (b - 3t)/t = (350 \text{ mm} - 3 \cdot 10 \text{ mm}) / 10 \text{ mm} = \mathbf{32.00} \rightarrow 38\epsilon = 30.92 < 32.00 \leq 42\epsilon = 34.17 \rightarrow \mathbf{\text{class 3}}$$

$$\text{Column 2: } c/t = (400 \text{ mm} - 3 \cdot 10 \text{ mm}) / 10 \text{ mm} = \mathbf{37.00} > 42\epsilon = 34.17 \rightarrow \mathbf{\text{class 4}}$$

For axial compression class 3 is resisted on the full gross area, exactly like classes 1 and 2 — the elastic limit is reached everywhere at once. Class 4 needs the effective section.

2.3 Effective area of the class 4 section (EN 1993-1-5 §4.4, each wall an internal element under uniform compression):

$$\psi = \sigma_2/\sigma_1 = 1 \rightarrow k_\sigma = 4 \text{ (Table 4.1); } \bar{b} = c = 400 \text{ mm} - 3 \cdot 10 \text{ mm} = 370 \text{ mm}$$

$$\bar{\lambda}_p = (\bar{b}/t) / (28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}) = (370 \text{ mm} / 10 \text{ mm}) / (28.4 \cdot 0.8136 \cdot \sqrt{4}) = \mathbf{0.8006} > 0.5 + \sqrt{(0.085 - 0.055\psi)} = 0.673 \quad (4.2)$$

$$\rho = (\bar{\lambda}_p - 0.055 \cdot (3 + \psi)) / \bar{\lambda}_p^2 = (0.8006 - 0.055 \cdot (3 + 1)) / 0.8006^2 = \mathbf{0.9058} (\leq 1)$$

$$\Delta A = 4 \cdot (1 - \rho) \cdot \bar{b} \cdot t = 4 \cdot (1 - 0.9058) \cdot 370 \text{ mm} \cdot 10 \text{ mm} = \mathbf{1394.1 \text{ mm}^2}$$

$$A_{\text{eff}} = A - \Delta A = 15256.6 \text{ mm}^2 - 1394.1 \text{ mm}^2 = \mathbf{13862.5 \text{ mm}^2} \quad (A_{\text{eff}}/A = 13862.5 / 15256.6 = 0.9086)$$

The section is doubly symmetric and all four walls lose the same strip, so the centroid does not move ($e_N = 0$) and the axial load causes no extra moment.

2.4 Flexural buckling (6.3.1; cold-formed hollow section $\rightarrow$ curve c, $\alpha = 0.49$; $\lambda_1 = \pi \cdot \sqrt{(E/f_y)} = \pi \cdot \sqrt{(210\,000 \text{ MPa} / 355 \text{ MPa})} = 76.409$; $L_{\text{cr}} = 1.0 \cdot L = 10000 \text{ mm}$; $\gamma_{M0} = \gamma_{M1} = 1.0$; the same about y and z):

$$\textbf{Column 1 (class 3): } \bar{\lambda} = (L_{\text{cr}}/i)/\lambda_1 = (10000 \text{ mm} / 137.845 \text{ mm}) / 76.409 = \mathbf{0.9494} \quad (6.50)$$

$$\Phi = 0.5 \cdot [1 + \alpha \cdot (\bar{\lambda} - 0.2) + \bar{\lambda}^2] = 0.5 \cdot [1 + 0.49 \cdot (0.9494 - 0.2) + 0.9494^2] = \mathbf{1.1343}$$

$$\chi = 1/(\Phi + \sqrt{(\Phi^2 - \bar{\lambda}^2)}) = 1 / (1.1343 + \sqrt{(1.1343^2 - 0.9494^2)}) = \mathbf{0.5698} (\leq 1) \quad (6.49)$$

$$N_{b,Rd} = \chi \cdot A \cdot f_y / \gamma_{M1} = 0.5698 \cdot 13256.6 \text{ mm}^2 \cdot 355 \text{ MPa} / 1.0 = \mathbf{2681.51 \text{ kN}} \rightarrow \eta = N_{Ed} / N_{b,Rd} = 2250 \text{ kN} / 2681.51 \text{ kN} = \mathbf{0.8391} \quad (6.47), (6.46)$$

$$\text{Cross-section: } N_{c,Rd} = A \cdot f_y / \gamma_{M0} = 13256.6 \text{ mm}^2 \cdot 355 \text{ MPa} / 1.0 = \mathbf{4706.11 \text{ kN}} \rightarrow \eta = 2250 \text{ kN} / 4706.11 \text{ kN} = \mathbf{0.4781} \quad (6.10), (6.9)$$

$$\textbf{Column 2 (class 4): } \bar{\lambda} = (L_{\text{cr}}/i)/\lambda_1 \cdot \sqrt{(A_{\text{eff}}/A)} = (10000 \text{ mm} / 158.268 \text{ mm}) / 76.409 \cdot \sqrt{(13862.5 \text{ mm}^2 / 15256.6 \text{ mm}^2)} = \mathbf{0.7882} \quad (6.51)$$

$$\Phi = 0.5 \cdot [1 + \alpha \cdot (\bar{\lambda} - 0.2) + \bar{\lambda}^2] = 0.5 \cdot [1 + 0.49 \cdot (0.7882 - 0.2) + 0.7882^2] = \mathbf{0.9548}$$

$$\chi = 1/(\Phi + \sqrt{(\Phi^2 - \bar{\lambda}^2)}) = 1 / (0.9548 + \sqrt{(0.9548^2 - 0.7882^2)}) = \mathbf{0.6695} (\leq 1) \quad (6.49)$$

$$N_{b,Rd} = \chi \cdot A_{\text{eff}} \cdot f_y / \gamma_{M1} = 0.6695 \cdot 13862.5 \text{ mm}^2 \cdot 355 \text{ MPa} / 1.0 = \mathbf{3294.98 \text{ kN}} \rightarrow \eta = N_{Ed} / N_{b,Rd} = 2250 \text{ kN} / 3294.98 \text{ kN} = \mathbf{0.6829} \quad (6.48), (6.46)$$

$$\text{Cross-section: } N_{c,Rd} = A_{\text{eff}} \cdot f_y / \gamma_{M0} = 13862.5 \text{ mm}^2 \cdot 355 \text{ MPa} / 1.0 = \mathbf{4921.20 \text{ kN}} \rightarrow \eta = 2250 \text{ kN} / 4921.20 \text{ kN} = \mathbf{0.4572} \quad (6.11), (6.9)$$

Local buckling costs column 2 9.1 % of its squash load, but its larger radius of gyration more than makes up for it: $N_{b,Rd}$ is 23 % higher than column 1's. The $\sqrt{(A_{\text{eff}}/A)}$ factor in (6.51) also lowers its slenderness, which slightly raises χ .

2.5 Torsional and flexural-torsional buckling (6.3.1.4). The clause matters for open thin-walled sections: angles, tees, channels and some I-sections, where a small torsional constant and a shear centre away from the centroid can make $N_{cr,T}$ lower than the flexural critical load. A closed square tube is the opposite case. Its shear centre is at the centroid, so the torsional mode cannot couple with the flexural one, and its warping constant is negligible ($I_w \approx 0$). With $G = 81$ GPa and $i_0^2 = i_y^2 + i_z^2$:

$$N_{cr,T} = (G \cdot I_t + \pi^2 \cdot E \cdot I_w / L_{cr,T}^2) / i_0^2 \approx G \cdot I_t / i_0^2$$

Column 1: $N_{cr,T} = G \cdot I_t / i_0^2 = 81\,000 \text{ MPa} \cdot 400\,828\,425 \text{ mm}^4 / 38\,002 \text{ mm}^2 = \mathbf{854\,346 \text{ kN}}$, $i_0^2 = 2 \cdot (137.845 \text{ mm})^2 = 38\,002 \text{ mm}^2$

flexural $N_{cr} = \pi^2 \cdot E \cdot I / L^2 = \pi^2 \cdot 210\,000 \text{ MPa} \cdot 251\,891\,369 \text{ mm}^4 / (10000 \text{ mm})^2 = 5\,221 \text{ kN}$;

$\bar{\lambda}_T = \sqrt{(N_{c,Rd} / N_{cr,T})} = \sqrt{(4706.11 \text{ kN} / 854\,346 \text{ kN})} = \mathbf{0.074}$

Column 2: $N_{cr,T} = G \cdot I_t / i_0^2 = 81\,000 \text{ MPa} \cdot 603\,804\,804 \text{ mm}^4 / 50\,098 \text{ mm}^2 = \mathbf{976\,260 \text{ kN}}$, $i_0^2 = 2 \cdot (158.268 \text{ mm})^2 = 50\,098 \text{ mm}^2$

flexural $N_{cr} = \pi^2 \cdot E \cdot I / L^2 = \pi^2 \cdot 210\,000 \text{ MPa} \cdot 382\,159\,879 \text{ mm}^4 / (10000 \text{ mm})^2 = 7\,921 \text{ kN}$;

$\bar{\lambda}_T = \sqrt{(N_{c,Rd} / N_{cr,T})} = \sqrt{(4921.20 \text{ kN} / 976\,260 \text{ kN})} = \mathbf{0.071}$

The torsional critical load is more than a hundred times the flexural one and $\bar{\lambda}_T < 0.2$, so $\chi_T = 1$ (6.3.1.2(4)). Torsional buckling cannot govern a hollow section, and EC3 does not ask for the check. The assessment follows this: closed sections carry no warping data, so it skips §6.3.1.4 for them and lists only the cross-section and flexural buckling checks.

3. FEM model and assessment settings

The structure is stored as a regular project file, `steel-column-class34.json`: two vertical members of 10 m, 5 m apart and unconnected, with MSH 350×350×10 and MSH 400×400×10 from the EN 10219 section library, S355. Each base node holds the three translations and the rotation about the column axis (torsion); each top node is held horizontally in x and y and free vertically. Two load cases, *LC1* (permanent, −1000 kN at each top) and *LC2* (imposed, −600 kN), self-weight off, and $CO1 = 1.35 \cdot LC1 + 1.5 \cdot LC2$. Forces and displacements come from the core solver (`analyze()`), the checks from the steel assessment (`assessSteel()`, *Assessment* → *Steel*) on *CO1* with the dialog's defaults: recommended $\gamma_{M0} = \gamma_{M1} = 1.0$, f_y from EN 1993-1-1 Table 3.1, buckling lengths $1.0 \cdot L$ about both axes. The buckling curve follows from the section itself (EN 10219, cold formed).

4. Results comparison

Quantity	Formula	Hand solution	FEM / assessment	Difference
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Support reactions, axial force and shortening (FEM, unrounded)				
R_z base, column 1 [kN] (CO1)	$1.35 \cdot G + 1.5 \cdot Q$	2250.0000	2250.0000000000000000	0
R_z base, column 2 [kN] (CO1)	$1.35 \cdot G + 1.5 \cdot Q$	2250.0000	2250.0000000000000000	0
N_{Ed} , column 1 / column 2 [kN]	equilibrium	2250.0000	2250.0000000000000000	0
Shortening, column 1, LC1 [mm]	$G \cdot L / (EA)$	3.59209	3.592091070938	0
Shortening, column 2, LC1 [mm]	$G \cdot L / (EA)$	3.12120	3.121202098948	≈ 0
Column 1 — MSH 350×350×10, class 3 (assessment working, 3 decimals)				
$N_{Rk} = A \cdot f_y$ [kN]	gross area	4706.106	4706.11	4×10^{-3} (rounding)
$\bar{\lambda}$	$L_{cr}/i/\lambda_1$	0.9494	0.949	4×10^{-4} (rounding)
Φ	(6.49)	1.1343	1.134	3×10^{-4} (rounding)
χ (curve c)	(6.49)	0.5698	0.570	2×10^{-4} (rounding)
$N_{b,Rd}$ [kN]	$\chi \cdot A \cdot f_y / \gamma_{M1}$	2681.51	2681.51	2×10^{-4} (rounding)
Column 2 — MSH 400×400×10, class 4 (assessment working, 2–3 decimals)				
A_{eff} [cm ²]	$A - 4(1 - \rho) \cdot c \cdot t$	138.6255	138.63	5×10^{-3} (rounding)
$N_{Rk} = A_{eff} \cdot f_y$ [kN]	effective area	4921.205	4921.20	5×10^{-3} (rounding)
$\bar{\lambda}$	$(L_{cr}/i/\lambda_1) \cdot \sqrt{A_{eff}/A}$	0.7882	0.788	2×10^{-4} (rounding)
Φ	(6.49)	0.9548	0.955	2×10^{-4} (rounding)
χ (curve c)	(6.49)	0.6695	0.670	5×10^{-4} (rounding)

$N_{b,Rd}$ [kN]	$\chi \cdot A_{eff} \cdot f_y / \gamma_{M1}$	3294.98	3294.98	4×10^{-4} (rounding)
Utilizations (assessment, unrounded)				
Column 1, compression 6.2.4	$N/(A \cdot f_y)$	0.4781	0.478102262258628	0
Column 1, buckling y / z 6.3.1	$N/N_{b,Rd}$	0.8391	0.839079546206083	0
Column 2, compression 6.2.4	$N/(A_{eff} \cdot f_y)$	0.4572	0.457205101145901	≈ 0
Column 2, buckling y / z 6.3.1	$N/N_{b,Rd}$	0.6829	0.682857025900780	≈ 0
Governing (overall)	max	0.8391	0.839079546206083	0

Forces in kN. The utilizations are unrounded and agree to floating-point round-off. The working the assessment prints (A_{eff} , N_{Rk} , λ , Φ , χ , $N_{b,Rd}$) is shown to 2–3 decimals; those rows agree to the last printed digit, and the differences shown are that rounding, not a calculation error. Buckling about y and about z give the same values for these square sections; one row stands for both.

✓ All quantities match; governing check flexural buckling of column 1, $\eta = 0.839$ **VERIFIED**

5. Final design value

For each column the largest design axial force it can carry is its lowest resistance, $N_{Rd,max} = \min(N_{c,Rd}, N_{b,Rd})$ — flexural buckling for both:

Column 1 (class 3): $N_{Rd,max} = \min(4706.11; 2681.51)$ kN = **2681.51 kN** ($\eta = 2250 / 2681.51 = 0.8391$)

Column 2 (class 4): $N_{Rd,max} = \min(4921.20; 3294.98)$ kN = **3294.98 kN** ($\eta = 2250 / 3294.98 = 0.6829$)

The assessment gives the utilizations unrounded, so its maximum design force follows exactly as N_{Ed} / η_{max} :

Final value	Formula	Hand solution	FEM / assessment	Difference
Column 1, η_{max}	$N_{Ed}/N_{b,Rd}$	0.8391	0.839079546206083	0
Column 1, $N_{Rd,max}$ [kN]	$\chi \cdot A \cdot f_y \mid N_{Ed}/\eta_{max}$	2681.5098	2681.5097688573	0

Column 2, $\eta_{\max}$	$N_{Ed}/N_{b,Rd}$	0.6829	0.682857025900780	≈ 0
Column 2, $N_{Rd,\max}$ [kN]	$\chi \cdot A_{eff} \cdot f_y \mid N_{Ed}/\eta_{\max}$	3294.9796	3294.9796438456	9×10^{-13}

✓ **Final design: column 1 $N_{Rd,\max} = 2681.51$ kN ($\eta = 0.8391$, governing), column 2 $N_{Rd,\max} = 3294.98$ kN ($\eta = 0.6829$) — hand and FEM agree** **VERIFIED**

6. Conclusion

The FEM calculator's steel assessment reproduces the EN 1993-1-1 / EN 1993-1-5 hand calculation of flexural buckling for a class 3 and a class 4 hollow-section column. It classifies both walls correctly, uses the gross area for class 3, and for class 4 finds the effective width of each wall and $A_{eff} = A - \Delta A$. It carries A_{eff} into both the resistance and the slenderness (6.51), and gives the same buckling resistance as the hand calculation. It correctly leaves out the torsional buckling check of §6.3.1.4, which cannot govern a closed section. The final design values $N_{Rd,\max}$ follow from its utilizations exactly as from the hand calculation.

The same comparison runs as part of the automated test suite on every test run:

`tests/verificationExamples.test.ts` solves the project file `calc_tests/verification/steel-column-class34.json`, runs the steel assessment on CO1 and asserts the support reactions, axial forces, shortening, the classes, ρ and A_{eff} , the buckling working, the absence of a torsional buckling check, every utilization and the final design values against the hand calculation above, recomputed from the sections and material stored in the file.