

Verification — Steel Beam, Lateral-Torsional Buckling (EC3)

FEM Calculator steel assessment (EN 1993-1-1) checked against a hand calculation: a simply supported IPE beam in bending about y, laterally unrestrained between its fork supports

Structure type	Simply supported beam IPE 300, S235, span 6 m, fork supports at both ends, no intermediate lateral restraint; uniform load on the top flange
Verification method	EN 1993-1-1 by hand: cross-section 6.2.5 and 6.2.6; lateral-torsional buckling 6.3.2.3 (rolled sections) with the modification factor f (6.58); M_{cr} from the three-factor formula with C_1 , C_2 and the load height z_g
Quantities checked	Support reactions, internal forces, deflections; C_1 , $C_2 \cdot z_g$, M_{cr} , λ_{LT} , Φ_{LT} , χ_{LT} , $\chi_{LT,mod}$, $M_{b,Rd}$; every utilization; the same beam with the load at the shear centre
Result	Exact match — forces, deflections and utilizations to $\approx 10^{-15}$; the working the report prints (2–3 decimals) equals the hand value to that rounding. Governing: LTB, $\eta = 0.976$, against 0.480 for the cross-section

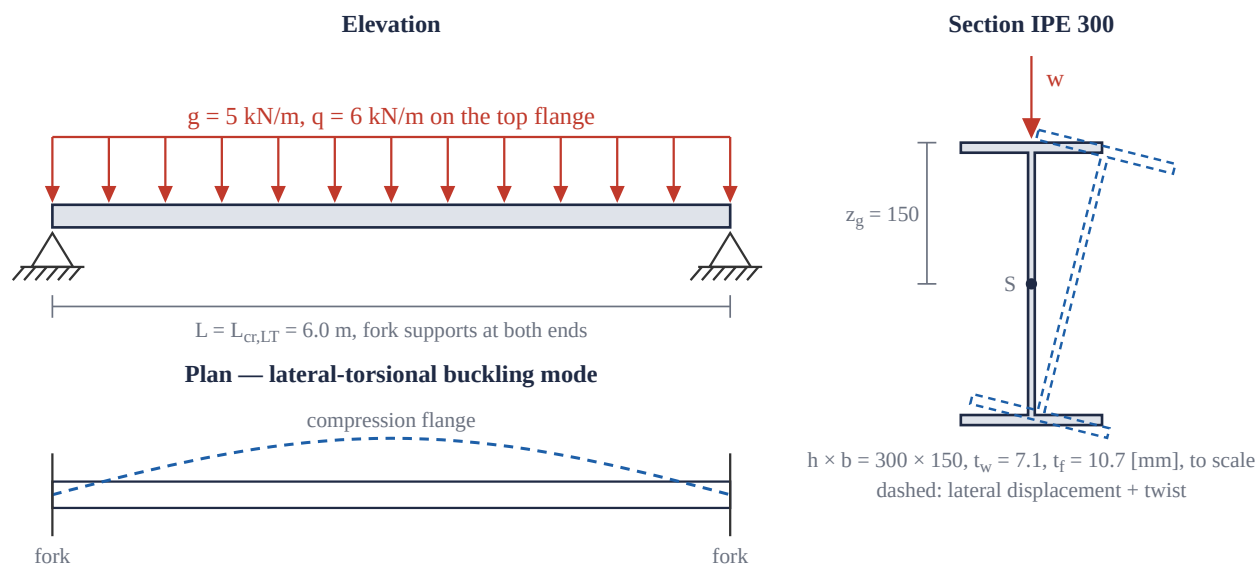


Fig. 1 — The beam, its lateral-torsional buckling mode in plan (the compression flange moves sideways between the forks) and the section, with the load applied on the top flange, z_g above the shear centre S .

1. Structure and loading

A floor beam IPE 300 in S235 ($f_y = 235 \text{ MPa}$ for $t_f = 10.7 \text{ mm} \leq 40 \text{ mm}$; $E = 210 \text{ GPa}$, $G = 81 \text{ GPa}$) spans $L = 6 \text{ m}$ between two supports. Both supports are **forks**: they hold the beam against lateral displacement and twist, but leave it free to rotate about both axes and to warp. Between the supports

nothing holds the compression flange sideways — no slab, no purlins — so the beam can fail by lateral-torsional buckling long before its cross-section reaches its plastic moment. It carries a permanent load $g = 5 \text{ kN/m}$ and an imposed load $q = 6 \text{ kN/m}$, both applied on the top flange; self-weight is not included. The ULS combination is $CO1 = 1.35 \cdot G + 1.5 \cdot Q$, giving $w_d = 15.75 \text{ kN/m}$.

Section properties (catalogue): $I_y = 8\,356 \text{ cm}^4$, $I_z = 604 \text{ cm}^4$, $I_t = 20.1 \text{ cm}^4$, $W_{el,y} = 557 \text{ cm}^3$, $W_{pl,y} = 628 \text{ cm}^3$, $A_{v,z} = 25.68 \text{ cm}^2$. The warping constant follows from the doubly symmetric I formula $I_w = I_z \cdot (h - t_f)^2 / 4 = 126,379 \cdot 10^6 \text{ mm}^6 = 126.38 \cdot 10^3 \text{ cm}^6$ (the catalogue's $125.9 \cdot 10^3 \text{ cm}^6$ to its rounding).

2. Hand solution

Units: forces in N or kN, lengths in mm or m, stresses in MPa = N/mm².

2.1 Forces and deflections.

$$w_d = 1.35 \cdot g + 1.5 \cdot q = 1.35 \cdot 5 \text{ kN/m} + 1.5 \cdot 6 \text{ kN/m} = \mathbf{15.75 \text{ kN/m}}$$

$$R_A = R_B = V_{Ed} = w_d \cdot L / 2 = 15.75 \text{ kN/m} \cdot 6 \text{ m} / 2 = \mathbf{47.250 \text{ kN}}$$

$$M_{Ed} = w_d \cdot L^2 / 8 = 15.75 \text{ kN/m} \cdot (6 \text{ m})^2 / 8 = \mathbf{70.8750 \text{ kNm}}$$

$$EI_y = 210\,000 \text{ MPa} \cdot 8\,356 \cdot 10^4 \text{ mm}^4 = 17\,548 \text{ kNm}^2$$

Midspan deflection (characteristic), $\delta = 5 \cdot w \cdot L^4 / (384 \cdot EI_y)$:

$$g: 5 \cdot 5 \text{ kN/m} \cdot (6 \text{ m})^4 / (384 \cdot 17\,548 \text{ kNm}^2) = \mathbf{4.8083 \text{ mm}}; \quad q: 5 \cdot 6 \text{ kN/m} \cdot (6 \text{ m})^4 / (384 \cdot 17\,548 \text{ kNm}^2) = \mathbf{5.7700 \text{ mm}}$$

2.2 Cross-section (Table 5.2, $\varepsilon = \sqrt{(235/f_y)} = \sqrt{(235 \text{ MPa} / 235 \text{ MPa})} = 1.0$; $\gamma_{M0} = 1.0$):

$$\text{Flange: } c/t = ((b - t_w)/2)/t_f = ((150 \text{ mm} - 7.1 \text{ mm})/2) / 10.7 \text{ mm} = 6.68 \leq 9\varepsilon = 9.00 \rightarrow \text{class 1}$$

$$\text{Web: } c/t = (h - 2 \cdot t_f)/t_w = (300 \text{ mm} - 2 \cdot 10.7 \text{ mm}) / 7.1 \text{ mm} = 39.24 \leq 72\varepsilon = 72.00 \rightarrow \text{class 1}$$

(root radii conservatively left in c)

$$M_{c,Rd} = M_{y,Rk} = W_{pl,y} \cdot f_y / \gamma_{M0} = 628\,000 \text{ mm}^3 \cdot 235 \text{ MPa} / 1.0 = \mathbf{147.58 \text{ kNm}} \rightarrow \eta = M_{Ed} / M_{c,Rd} = 70.8750 \text{ kNm} / 147.58 \text{ kNm} = \mathbf{0.4802} \quad (6.12)$$

$$V_{pl,z,Rd} = A_{v,z} \cdot f_y / (\sqrt{3} \cdot \gamma_{M0}) = 2568.2 \text{ mm}^2 \cdot 235 \text{ MPa} / (\sqrt{3} \cdot 1.0) = \mathbf{348.44 \text{ kN}} \rightarrow \eta = 47.250 \text{ kN} / 348.44 \text{ kN} = \mathbf{0.1356} (< 0.5, \text{ no M-V interaction}) \quad (6.17)$$

2.3 Elastic critical moment M_{cr} . For a fork-supported beam with the load at height z_g above the shear centre, the three-factor formula (doubly symmetric section, $z_j = 0$, $k = k_w = 1$) is

$$M_{cr} = C_1 \cdot \pi^2 EI_z / L^2 \cdot [\sqrt{ I_w / I_z + L^2 GI_t / (\pi^2 EI_z) + (C_2 z_g)^2 } - C_2 z_g] \quad (1)$$

$$I_w = I_z \cdot (h - t_f)^2 / 4 = 604 \cdot 10^4 \text{ mm}^4 \cdot (300 \text{ mm} - 10.7 \text{ mm})^2 / 4 = \mathbf{1.2638 \cdot 10^{11} \text{ mm}^6}$$

C_1 from the moment diagram (López, Yong & Serna 2006): with $M_2 = M_4 = 0.75 \cdot M_{max}$, $M_3 = M_{max}$ at the quarter points of the parabola,

$$A_1 = (M_{max}^2 + 5 \cdot M_2^2 + 10 \cdot M_3^2 + 5 \cdot M_4^2) / (21 \cdot M_{max}^2) = (1 + 5 \cdot 0.75^2 + 10 \cdot 1^2 + 5 \cdot 0.75^2) / 21 = 0.79167$$

$$C_1 = 1 / \sqrt{A_1} = 1 / \sqrt{0.79167} = \mathbf{1.1239} \text{ (tables: 1.127–1.13)}$$

$$C_2 = 0.454 \text{ (UDL)}; z_g = I_y / W_{el,y} = 8 \, 356 \cdot 10^4 \text{ mm}^4 / 557 \cdot 10^3 \text{ mm}^3 = 150.018 \text{ mm (to the loaded flange)} \rightarrow C_2 \cdot z_g = 0.454 \cdot 150.018 \text{ mm} = \mathbf{68.108 \text{ mm}}$$

$$\pi^2 EI_z / L^2 = \pi^2 \cdot 210 \, 000 \text{ MPa} \cdot 604 \cdot 10^4 \text{ mm}^4 / (6 \, 000 \text{ mm})^2 = \mathbf{347 \, 739 \text{ N}}$$

$$I_w / I_z = 1.2638 \cdot 10^{11} \text{ mm}^6 / 604 \cdot 10^4 \text{ mm}^4 = 20 \, 924 \text{ mm}^2$$

$$L^2 GI_t / (\pi^2 EI_z) = (6 \, 000 \text{ mm})^2 \cdot 81 \, 000 \text{ MPa} \cdot 20.1 \cdot 10^4 \text{ mm}^4 / (\pi^2 \cdot 210 \, 000 \text{ MPa} \cdot 604 \cdot 10^4 \text{ mm}^4) = 46 \, 820 \text{ mm}^2$$

$$(C_2 z_g)^2 = (68.108 \text{ mm})^2 = 4 \, 639 \text{ mm}^2$$

$$\text{Top flange: } M_{cr} = 1.1239 \cdot 347 \, 739 \text{ N} \cdot [\sqrt{(20 \, 924 + 46 \, 820 + 4 \, 639) \text{ mm}} - 68.108 \text{ mm}] = 1.1239 \cdot 347 \, 739 \text{ N} \cdot (269.039 - 68.108) \text{ mm} = \mathbf{78.529 \text{ kNm}}$$

$$\text{Shear centre } (z_g = 0): M_{cr} = 1.1239 \cdot 347 \, 739 \text{ N} \cdot \sqrt{(20 \, 924 + 46 \, 820) \text{ mm}} = 1.1239 \cdot 347 \, 739 \text{ N} \cdot 260.275 \text{ mm} = \mathbf{101.722 \text{ kNm}}$$

Applying the load on the top flange instead of at the shear centre lowers M_{cr} by 23 %: a load above the shear centre twists the beam further as it displaces sideways.

2.4 Buckling resistance (6.3.2.3, rolled I with $h/b = 300/150 = 2.0 \leq 2 \rightarrow$ curve b, $\alpha_{LT} = 0.34$; $\bar{\lambda}_{LT,0} = 0.4$, $\beta = 0.75$; $\gamma_{M1} = 1.0$; $k_c = 1/\sqrt{C_1} = 1/\sqrt{1.1239} = 0.9433$, Table 6.6: 0.94):

$$\text{Load on the top flange: } \bar{\lambda}_{LT} = \sqrt{(M_{y,Rk} / M_{cr})} = \sqrt{(147.58 \text{ kNm} / 78.529 \text{ kNm})} = \mathbf{1.3709} \quad (6.56)$$

$$\Phi_{LT} = 0.5 \cdot [1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - 0.4) + \beta \cdot \bar{\lambda}_{LT}^2] = 0.5 \cdot [1 + 0.34 \cdot (1.3709 - 0.4) + 0.75 \cdot 1.3709^2] = \mathbf{1.3698}$$

$$\chi_{LT} = 1 / (\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2)}) = 1 / (1.3698 + \sqrt{(1.3698^2 - 0.75 \cdot 1.3709^2)}) = \mathbf{0.4871} \quad (\leq 1, \leq 1/\bar{\lambda}_{LT}^2 = 1/1.3709^2 = 0.5321) \quad (6.57)$$

$$f = 1 - 0.5 \cdot (1 - k_c) \cdot [1 - 2 \cdot (\bar{\lambda}_{LT} - 0.8)^2] = 1 - 0.5 \cdot (1 - 0.9433) \cdot [1 - 2 \cdot (1.3709 - 0.8)^2] = \mathbf{0.9901}$$

$$\chi_{LT,mod} = \chi_{LT} / f = 0.4871 / 0.9901 = \mathbf{0.4919} \quad (\leq 1, \leq 1/\bar{\lambda}_{LT}^2) \quad (6.58)$$

$$M_{b,Rd} = \chi_{LT,mod} \cdot M_{y,Rk} / \gamma_{M1} = 0.4919 \cdot 147.58 \text{ kNm} / 1.0 = \mathbf{72.600 \text{ kNm}} \rightarrow \eta = M_{Ed} / M_{b,Rd} = 70.8750 \text{ kNm} / 72.600 \text{ kNm} = \mathbf{0.9762} \quad (6.54)$$

Load at the shear centre: $\bar{\lambda}_{LT} = \sqrt{(M_{y,Rk} / M_{cr})} = \sqrt{(147.58 \text{ kNm} / 101.722 \text{ kNm})} = \mathbf{1.2045} \quad (6.56)$

$$\Phi_{LT} = 0.5 \cdot [1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - 0.4) + \beta \cdot \bar{\lambda}_{LT}^2] = 0.5 \cdot [1 + 0.34 \cdot (1.2045 - 0.4) + 0.75 \cdot 1.2045^2] = \mathbf{1.1808}$$

$$\chi_{LT} = 1 / (\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2)}) = 1 / (1.1808 + \sqrt{(1.1808^2 - 0.75 \cdot 1.2045^2)}) = \mathbf{0.5766} \quad (\leq 1, \leq 1 / \bar{\lambda}_{LT}^2 = 1 / 1.2045^2 = 0.6893) \quad (6.57)$$

$$f = 1 - 0.5 \cdot (1 - k_c) \cdot [1 - 2 \cdot (\bar{\lambda}_{LT} - 0.8)^2] = 1 - 0.5 \cdot (1 - 0.9433) \cdot [1 - 2 \cdot (1.2045 - 0.8)^2] = \mathbf{0.9809}$$

$$\chi_{LT,mod} = \chi_{LT} / f = 0.5766 / 0.9809 = \mathbf{0.5879} \quad (\leq 1, \leq 1 / \bar{\lambda}_{LT}^2) \quad (6.58)$$

$$M_{b,Rd} = \chi_{LT,mod} \cdot M_{y,Rk} / \gamma_{M1} = 0.5879 \cdot 147.58 \text{ kNm} / 1.0 = \mathbf{86.755 \text{ kNm}} \rightarrow \eta = M_{Ed} / M_{b,Rd} = 70.8750 \text{ kNm} / 86.755 \text{ kNm} = \mathbf{0.8170} \quad (6.54)$$

Lateral-torsional buckling governs: the beam can carry only 49 % of its plastic moment. It still passes with the load on the top flange ($\eta = 0.976$); the same load at the shear centre would leave a 18 % margin.

3. FEM model and assessment settings

The structure is stored as a regular project file, `steel-beam-ltb.json`: one member of 6 m along global x, IPE 300 from the section library, S235. Node 1 holds all three translations and the torsional rotation r_x ; node 2 the same except the axial translation — the forks; bending rotations are free at both ends. Two load cases, *LC1* (permanent, $g = 5 \text{ kN/m}$) and *LC2* (imposed, $q = 6 \text{ kN/m}$), as full-span global distributed loads in $-Z$, self-weight off, and $CO1 = 1.35 \cdot LC1 + 1.5 \cdot LC2$. Forces and deflections come from the core solver (`analyze()`), the checks from the steel assessment (`assessSteel()`), *Assessment* $\rightarrow$ *Steel*) on *CO1* with the dialog's defaults: recommended $\gamma_{M0} = \gamma_{M1} = 1.0$, C_1 from the member's diagram, load on the top flange, $L_{cr,LT} = 1.0 \cdot L$. The shear-centre variant is the same run with *load position* set to the shear centre.

4. Results comparison

Quantity	Formula	Hand solution	FEM / assessment	Difference
Support reactions, internal forces and deflections (FEM, unrounded)				
$R_A = R_B \text{ [kN]} (CO1)$	$w_d \cdot L / 2$	47.2500	47.249999999999986	1×10^{-14}

$M_{y,Ed}$ at midspan [kNm]	$w_d \cdot L^2/8$	70.8750	70.87499999999957	4×10^{-14}
$V_{z,Ed}$ at the support [kN]	$w_d \cdot L/2$	47.2500	47.24999999999986	1×10^{-14}
Midspan deflection, LC1 g [mm]	$5gL^4/(384EI_y)$	4.8083	4.808349859810	≈ 0
Midspan deflection, LC2 q [mm]	$5qL^4/(384EI_y)$	5.7700	5.770019831772	≈ 0
LTB working, load on the top flange (as the assessment shows it, 2–3 decimals)				
C_1 from the diagram	$1/\sqrt{A_1}$	1.1239	1.124	1×10^{-4} (rounding)
$C_2 \cdot z_g$ [mm]	$0.454 \cdot I_y/W_{el,y}$	68.108	68.1	8×10^{-3} (rounding)
$M_{y,Rk}$ [kNm]	$W_{pl,y} \cdot f_y$	147.5800	147.58	0
M_{cr} [kNm]	three-factor	78.5288	78.53	1×10^{-3} (rounding)
$\bar{\lambda}_{LT}$	$\sqrt{(M_{y,Rk}/M_{cr})}$	1.3709	1.371	1×10^{-4} (rounding)
Φ_{LT}	(6.57)	1.3698	1.370	2×10^{-4} (rounding)
χ_{LT}	(6.57)	0.4871	0.487	8×10^{-5} (rounding)
$\chi_{LT,mod} = \chi_{LT}/f$	(6.58)	0.4919	0.492	6×10^{-5} (rounding)
$M_{b,Rd}$ [kNm]	$\chi_{LT,mod} \cdot M_{y,Rk}/\gamma_{M1}$	72.6003	72.60	3×10^{-4} (rounding)
LTB working, load at the shear centre ($z_g = 0$)				
M_{cr} [kNm]	two-factor	101.7221	101.72	2×10^{-3} (rounding)
$\bar{\lambda}_{LT}$	$\sqrt{(M_{y,Rk}/M_{cr})}$	1.2045	1.204	5×10^{-4} (rounding)
$\chi_{LT,mod}$	(6.57), (6.58)	0.5879	0.588	1×10^{-4} (rounding)
Utilizations at midspan (assessment, unrounded)				
Bending, 6.2.5	$M_{Ed}/M_{c,Rd}$	0.4802	0.480248001084157	≈ 0

Shear at the support, 6.2.6	$V_{Ed}/V_{pl,Rd}$	0.1356	0.135603384749179	≈ 0
LTB, load on the top flange, 6.3.2	$M_{Ed}/M_{b,Rd}$	0.9762	0.976235954600701	≈ 0
LTB, load at the shear centre, 6.3.2	$M_{Ed}/M_{b,Rd}$	0.8170	0.816951345071570	≈ 0
Governing (overall, top flange)	max	0.9762	0.976235954600701	≈ 0

Forces in kN, moments in kNm. The utilizations are unrounded and agree to floating-point round-off. The working the assessment prints (C_1 , M_{cr} , λ_{LT} , χ_{LT} , ...) is shown to 1–3 decimals; those rows agree to the last printed digit, and the differences shown are that rounding, not a calculation error.

✓ All quantities match; governing check lateral-torsional buckling, $\eta = 0.976$ **VERIFIED**

5. Final design value

The governing check is lateral-torsional buckling with the load on the top flange (6.54). Its design resistance moment and the largest uniform design load the beam can carry, $w_{d,max} = 8 \cdot M_{b,Rd}/L^2$:

$$M_{b,Rd} = 0.4919 \cdot 147.58 \text{ kNm} / 1.0 = \mathbf{72.6003 \text{ kNm}} \rightarrow w_{d,max} = 8 \cdot 72.6003 \text{ kNm} / (6 \text{ m})^2 = \mathbf{16.1334 \text{ kN/m}}$$

$$\text{Load at the shear centre, for comparison: } M_{b,Rd} = \mathbf{86.7555 \text{ kNm}} \rightarrow w_{d,max} = 8 \cdot 86.7555 \text{ kNm} / (6 \text{ m})^2 = \mathbf{19.2790 \text{ kN/m}}$$

The assessment gives the utilization unrounded, so its resistance follows exactly as M_{Ed}/η and its maximum load as w_d/η :

Final value	Formula	Hand solution	FEM / assessment	Difference
η , LTB, top flange (6.54)	$M_{Ed}/M_{b,Rd}$	0.9762	0.976235954600701	≈ 0
$M_{b,Rd}$, top flange [kNm]	$\chi_{LT,mod} \cdot M_{y,Rk} \mid M_{Ed}/\eta$	72.6003	72.600276261070	1×10^{-14}
$w_{d,max}$, top flange [kN/m]	$8 \cdot M_{b,Rd}/L^2 \mid w_d/\eta$	16.1334	16.133394724682	≈ 0
$M_{b,Rd}$, shear centre [kNm]	$\chi_{LT,mod} \cdot M_{y,Rk} \mid M_{Ed}/\eta$	86.7555	86.755472559741	1×10^{-14}
$w_{d,max}$, shear centre [kN/m]	$8 \cdot M_{b,Rd}/L^2 \mid w_d/\eta$	19.2790	19.278993902165	1×10^{-14}

✓ Final design: $\eta = 0.9762$, $M_{b,Rd} = 72.60 \text{ kNm}$, $w_{d,max} = 16.13 \text{ kN/m}$ (load on the top flange) — hand and FEM agree **VERIFIED**

6. Conclusion

The FEM calculator's steel assessment reproduces the EN 1993-1-1 hand calculation of lateral-torsional buckling of a fork-supported IPE beam: C_1 read off the member's own moment diagram, the load-height term $C_2 \cdot z_g$ for a load on the top flange, the elastic critical moment, the rolled-section curve of 6.3.2.3 with its modification factor f , and the resulting buckling resistance and utilization — for the load on the top flange and at the shear centre alike — and with it the final design values $M_{b,Rd}$ and $w_{d,max}$. Support reactions, internal forces and deflections match the beam formulas to round-off.

The same comparison runs as part of the automated test suite on every test run:

`tests/verificationExamples.test.ts` solves the project file `calc_tests/verification/steel-beam-ltb.json`, runs the steel assessment on CO1 with the load on the top flange and at the shear centre, and asserts the support reactions, deflections, internal forces, the LTB working, every utilization and the final design values against the hand calculation above, recomputed from the section and material stored in the file.