

Verification — Steel Angle Strut, Flexural-Torsional Buckling (EC3)

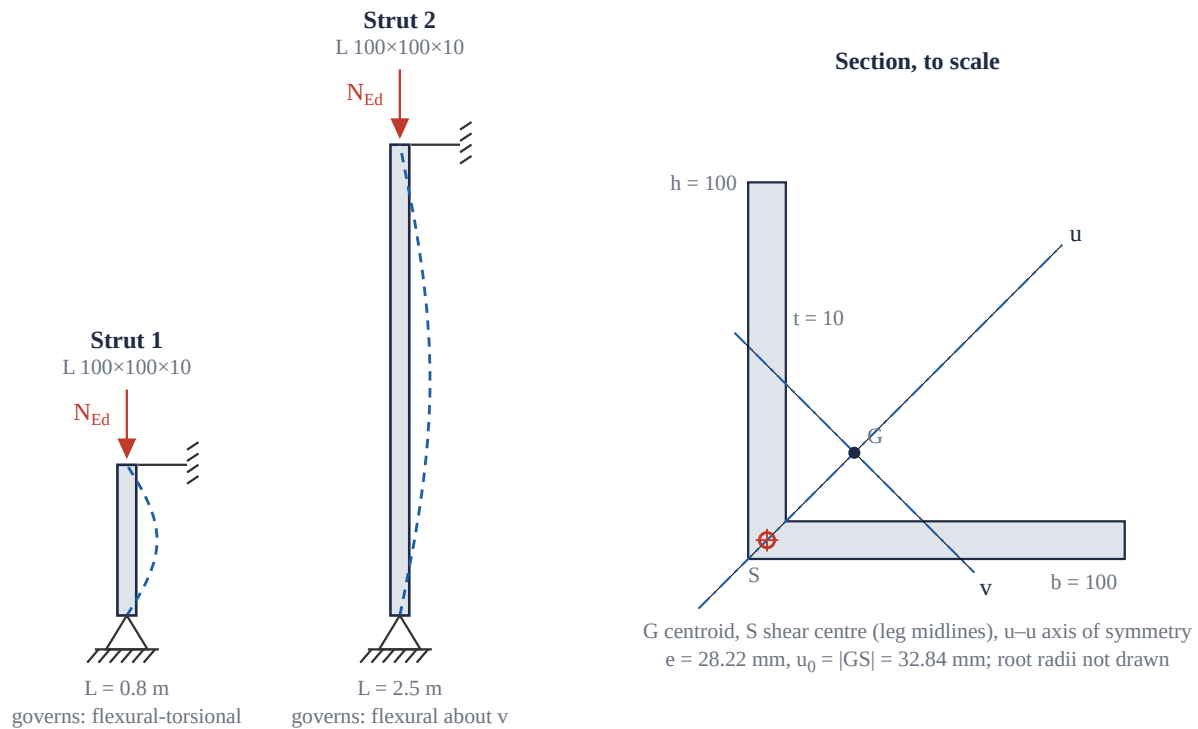
FEM Calculator steel assessment (EN 1993-1-1) checked against a hand calculation: an open, singly symmetric section — the equal angle L 100×100×10 — in compression, with flexural buckling about the principal axes u and v and torsional-flexural buckling by §6.3.1.4

Structure type	Two pinned struts, $L = 0.8$ m and $L = 2.5$ m, equal angle L 100×100×10 (EN 10056-1), S235, loaded concentrically through the centroid; ends held against twisting
Verification method	EN 1993-1-1 by hand: classification Table 5.2 (angle), compression 6.2.4, flexural buckling 6.3.1.2 about u–u and v–v, torsional and flexural-torsional buckling 6.3.1.4 with $N_{cr,T}$ and the coupled $N_{cr,TF}$ of a singly symmetric section; curve b throughout (Table 6.2, angles)
Quantities checked	Support reactions, axial force, axial shortening; class; i_0 , $N_{cr,T}$, $N_{cr,u}$, $N_{cr,TF}$; λ , χ and $N_{b,Rd}$ for each mode; every utilization; the maximum design force of each strut
Result	Exact match — forces, shortening and utilizations to $\approx 10^{-15}$; the working the report prints (1–3 decimals) equals the hand value to that rounding. Strut 1 is governed by flexural-torsional buckling ($\eta = 0.359$), strut 2 by flexural buckling about v ($\eta = 0.788$)
Maximum design force	$N_{Rd,max} = 392.68$ kN (strut 1) and 178.91 kN (strut 2), by hand and FEM alike

1. Structure and loading

Two struts of the same equal angle L 100×100×10 in S235 ($f_y = 235$ MPa for $t \leq 40$ mm, $E = 210$ GPa, $G = 81$ GPa, $\varepsilon = 1.0$): a short one, $L = 0.8$ m, and a long one, $L = 2.5$ m. Both are pinned at the ends, held laterally at both ends in both directions and held against twisting there (fork supports), so $L_{cr,u} = L_{cr,v} = L_{cr,T} = L$. Each carries a permanent load $G = 60$ kN and an imposed load $Q = 40$ kN applied through the centroid; self-weight is not included. $CO1 = 1.35 \cdot G + 1.5 \cdot Q$.

An angle is the textbook case for §6.3.1.4. Its two thin legs meet at a point, so its warping constant is practically zero and its torsional stiffness is small. Its shear centre, at the intersection of the leg midlines, lies well away from the centroid along the axis of symmetry. Torsion therefore couples with bending about that axis. The pure torsional critical load does not depend on the length, while the flexural ones fall with its square. The two struts are chosen so that each mode governs one of them.



2. Hand solution

Units: forces in N or kN, lengths in mm, stresses in MPa = N/mm².

2.1 Design force and shortening.

$$N_{Ed} = 1.35 \cdot G + 1.5 \cdot Q = 1.35 \cdot 60 \text{ kN} + 1.5 \cdot 40 \text{ kN} = \mathbf{141 \text{ kN}}$$
 in each strut

Shortening under G alone, $\Delta = G \cdot L / (E \cdot A)$: strut 1: $60\,000 \text{ N} \cdot 800 \text{ mm} / (210\,000 \text{ MPa} \cdot 1915.5 \text{ mm}^2) = \mathbf{0.11932 \text{ mm}}$; strut 2: $60\,000 \text{ N} \cdot 2\,500 \text{ mm} / (210\,000 \text{ MPa} \cdot 1915.5 \text{ mm}^2) = \mathbf{0.37289 \text{ mm}}$

2.2 Section properties (section library, EN 10056-1 dimensions with root radius $r_1 = 12 \text{ mm}$ and toe radius $r_2 = 6 \text{ mm}$). The legs are parallel to y and z; the principal axes are at $\alpha = 45^\circ$: u–u, the axis of symmetry through the heel, and v–v across it.

Section library: $A = 1915.5 \text{ mm}^2$, $I_y = I_z = 176.66 \text{ cm}^4$, $e = 28.22 \text{ mm}$ (centroid from the back of each leg)

$$i_u = 38.25 \text{ mm}, i_v = 19.52 \text{ mm} \text{ (section library)} \rightarrow I_u = i_u^2 \cdot A = (38.25 \text{ mm})^2 \cdot 1915.5 \text{ mm}^2 = \mathbf{2\,803\,109 \text{ mm}^4}$$

$$I_v = (19.52 \text{ mm})^2 \cdot 1915.5 \text{ mm}^2 = 730\,015 \text{ mm}^4$$

$$I_t = 2 \cdot (b - t/2) \cdot t^3 / 3 = 2 \cdot 95 \cdot 10^3 / 3 = \mathbf{63\,333 \text{ mm}^4}$$

$$I_w \approx 0 \text{ (thin legs meeting at the shear centre)}$$

Shear centre S at the leg midlines: $y_0 = z_0 = e - t/2 = 23.22$ mm from the centroid, i.e. $u_0 = \sqrt{2} \cdot 23.22 = \mathbf{32.84}$ mm along u-u, $v_0 = 0$

$$i_0^2 = i_u^2 + i_v^2 + y_0^2 + z_0^2 = 38.25^2 + 19.52^2 + 23.22^2 + 23.22^2 = 2\,923 \text{ mm}^2 \rightarrow i_0 = \mathbf{54.06 \text{ mm}}$$

2.3 Classification (Table 5.2, sheet 3, angle in compression; an angle has no class 1 or 2 here):

$$\varepsilon = \sqrt{(235 \text{ MPa} / 235 \text{ MPa})} = 1.0; \quad h/t = 100 \text{ mm} / 10 \text{ mm} = 10.00 \leq 15\varepsilon = 15.00; \quad (b + h)/(2t) = (100 \text{ mm} + 100 \text{ mm}) / (2 \cdot 10 \text{ mm}) = 10.00 \leq 11.5\varepsilon = 11.50 \rightarrow \mathbf{\text{class 3}}$$

$$N_{Rk} = N_{c,Rd} = A \cdot f_y / \gamma_{M0} = 1915.5 \text{ mm}^2 \cdot 235 \text{ MPa} / 1.0 = \mathbf{450.15 \text{ kN}} \rightarrow \eta = N_{Ed} / N_{c,Rd} = 141 \text{ kN} / 450.15 \text{ kN} = \mathbf{0.3132} \quad (6.10)$$

2.4 Flexural buckling (6.3.1.2, curve b, $\alpha = 0.34$; $\lambda_1 = \pi \cdot \sqrt{(E/f_y)} = \pi \cdot \sqrt{(210\,000 \text{ MPa} / 235 \text{ MPa})} = 93.913$; $L_{cr} = 1.0 \cdot L$; $\gamma_{M1} = 1.0$). The buckling modes of an angle are about its principal axes, not about the legs:

Strut 1, L = 0.8 m:

$$\bar{\lambda}_u = (L_{cr}/i_u)/\lambda_1 = (800 \text{ mm} / 38.25 \text{ mm}) / 93.913 = \mathbf{0.2227}; \quad \Phi_u = 0.5 \cdot [1 + 0.34 \cdot (0.2227 - 0.2) + 0.2227^2] = 0.5287 \quad (6.50)$$

$$\chi_u = 1 / (0.5287 + \sqrt{(0.5287^2 - 0.2227^2)}) = \mathbf{0.9920}; \quad N_{b,u,Rd} = \chi_u \cdot N_{Rk} / \gamma_{M1} = 0.9920 \cdot 450.15 \text{ kN} / 1.0 = \mathbf{446.53 \text{ kN}} \rightarrow \eta = 141 / 446.53 = \mathbf{0.3158}$$

$$\bar{\lambda}_v = (L_{cr}/i_v)/\lambda_1 = (800 \text{ mm} / 19.52 \text{ mm}) / 93.913 = \mathbf{0.4364}; \quad \Phi_v = 0.5 \cdot [1 + 0.34 \cdot (0.4364 - 0.2) + 0.4364^2] = 0.6354 \quad (6.50)$$

$$\chi_v = 1 / (0.6354 + \sqrt{(0.6354^2 - 0.4364^2)}) = \mathbf{0.9114}; \quad N_{b,v,Rd} = \chi_v \cdot N_{Rk} / \gamma_{M1} = 0.9114 \cdot 450.15 \text{ kN} / 1.0 = \mathbf{410.26 \text{ kN}} \rightarrow \eta = 141 / 410.26 = \mathbf{0.3437}$$

Strut 2, L = 2.5 m:

$$\bar{\lambda}_u = (L_{cr}/i_u)/\lambda_1 = (2\,500 \text{ mm} / 38.25 \text{ mm}) / 93.913 = \mathbf{0.6959}; \quad \Phi_u = 0.5 \cdot [1 + 0.34 \cdot (0.6959 - 0.2) + 0.6959^2] = 0.8264 \quad (6.50)$$

$$\chi_u = 1 / (0.8264 + \sqrt{(0.8264^2 - 0.6959^2)}) = \mathbf{0.7860}; \quad N_{b,u,Rd} = \chi_u \cdot N_{Rk} / \gamma_{M1} = 0.7860 \cdot 450.15 \text{ kN} / 1.0 = \mathbf{353.83 \text{ kN}} \rightarrow \eta = 141 / 353.83 = \mathbf{0.3985}$$

$$\bar{\lambda}_v = (L_{cr}/i_v)/\lambda_1 = (2\,500 \text{ mm} / 19.52 \text{ mm}) / 93.913 = \mathbf{1.3636}; \quad \Phi_v = 0.5 \cdot [1 + 0.34 \cdot (1.3636 - 0.2) + 1.3636^2] = 1.6276 \quad (6.50)$$

$$\chi_v = 1 / (1.6276 + \sqrt{(1.6276^2 - 1.3636^2)}) = \mathbf{0.3974}; \quad N_{b,v,Rd} = \chi_v \cdot N_{Rk} / \gamma_{M1} = 0.3974 \cdot 450.15 \text{ kN} / 1.0 = \mathbf{178.91 \text{ kN}} \rightarrow \eta = 141 / 178.91 = \mathbf{0.7881}$$

2.5 Torsional and flexural-torsional buckling (6.3.1.4). The pure torsional critical load, with $I_w = 0$, does not depend on the length:

$$N_{cr,T} = (G \cdot I_t + \pi^2 \cdot E \cdot I_w / L_{cr,T}^2) / i_0^2 = (81\,000 \text{ MPa} \cdot 63\,333 \text{ mm}^4 + 0) / 2\,923 \text{ mm}^2 = \mathbf{1755.06 \text{ kN}}$$

The shear centre is on u–u, so torsion couples with flexure about u–u; the flexural mode about v–v stays uncoupled. With $\beta = 1 - (u_0/i_0)^2 = 1 - (32.84 \text{ mm} / 54.06 \text{ mm})^2 = \mathbf{0.6310}$, $N_{cr,TF}$ is the lower root of $\beta \cdot N^2 - (N_{cr,u} + N_{cr,T}) \cdot N + N_{cr,u} \cdot N_{cr,T} = 0$:

$$N_{cr,TF} = [(N_{cr,u} + N_{cr,T}) - \sqrt{(N_{cr,u} + N_{cr,T})^2 - 4\beta \cdot N_{cr,u} \cdot N_{cr,T}}] / (2\beta); \quad \bar{\chi}_T = \sqrt{(N_{Rk} / N_{cr,TF})},$$

curve b

Strut 1, L = 0.8 m: $N_{cr,u} = \pi^2 \cdot E \cdot I_u / L^2 = \pi^2 \cdot 210\,000 \text{ MPa} \cdot 2\,803\,109 \text{ mm}^4 / (800 \text{ mm})^2 = \mathbf{9077.77 \text{ kN}}$

$$N_{cr,TF} = [(9077.77 + 1755.06) - \sqrt{(9077.77 + 1755.06)^2 - 4 \cdot 0.6310 \cdot 9077.77 \cdot 1755.06}] \text{ kN} / (2 \cdot 0.6310) = \mathbf{1624.42 \text{ kN}}$$

$$\bar{\chi}_T = \sqrt{(N_{Rk} / N_{cr,TF})} = \sqrt{(450.15 \text{ kN} / 1624.42 \text{ kN})} = \mathbf{0.5264}; \quad \Phi_T = 0.5 \cdot [1 + 0.34 \cdot (0.5264 - 0.2) + 0.5264^2] = 0.6940$$

$$\chi_T = 1 / (0.6940 + \sqrt{(0.6940^2 - 0.5264^2)}) = \mathbf{0.8723}; \quad N_{b,T,Rd} = 0.8723 \cdot 450.15 \text{ kN} / 1.0 = \mathbf{392.68 \text{ kN}} \rightarrow \eta = 141 / 392.68 = \mathbf{0.3591}$$

Strut 2, L = 2.5 m: $N_{cr,u} = \pi^2 \cdot E \cdot I_u / L^2 = \pi^2 \cdot 210\,000 \text{ MPa} \cdot 2\,803\,109 \text{ mm}^4 / (2\,500 \text{ mm})^2 = \mathbf{929.56 \text{ kN}}$

$$N_{cr,TF} = [(929.56 + 1755.06) - \sqrt{(929.56 + 1755.06)^2 - 4 \cdot 0.6310 \cdot 929.56 \cdot 1755.06}] \text{ kN} / (2 \cdot 0.6310) = \mathbf{734.51 \text{ kN}}$$

$$\bar{\chi}_T = \sqrt{(N_{Rk} / N_{cr,TF})} = \sqrt{(450.15 \text{ kN} / 734.51 \text{ kN})} = \mathbf{0.7829}; \quad \Phi_T = 0.5 \cdot [1 + 0.34 \cdot (0.7829 - 0.2) + 0.7829^2] = 0.9055$$

$$\chi_T = 1 / (0.9055 + \sqrt{(0.9055^2 - 0.7829^2)}) = \mathbf{0.7350}; \quad N_{b,T,Rd} = 0.7350 \cdot 450.15 \text{ kN} / 1.0 = \mathbf{330.85 \text{ kN}} \rightarrow \eta = 141 / 330.85 = \mathbf{0.4262}$$

$N_{cr,TF}$ is lower than both $N_{cr,T}$ and $N_{cr,u}$, which is what coupling does. In the short strut $N_{cr,u}$ is large, so $N_{cr,TF}$ stays close to the length-independent $N_{cr,T}$: 1624 kN against $N_{cr,v} = 2364 \text{ kN}$, so flexural-torsional buckling governs ($\eta = 0.3591 > 0.3437$). In the long strut v–v flexure falls much faster ($N_{cr,v} = 242 \text{ kN}$ against $N_{cr,TF} = 735 \text{ kN}$) and governs ($\eta = 0.7881$).

3. FEM model and assessment settings

The structure is stored as a regular project file, `steel-angle-strut.json`: two vertical members, 0.8 m and 2.5 m long, 2 m apart and unconnected, both L 100×100×10 from the EN 10056-1 section library,

S235. Each base node holds the three translations and the rotation about the member axis; each top node is held horizontally in x and y and against that rotation, and is free vertically. Two load cases, *LC1* (permanent, −60 kN at each top) and *LC2* (imposed, −40 kN), self-weight off, and $CO1 = 1.35 \cdot LC1 + 1.5 \cdot LC2$. Forces and displacements come from the core solver (`analyze()`), the checks from the steel assessment (`assessSteel()` , *Assessment* → *Steel*) on CO1 with the dialog's defaults: recommended $\gamma_{M0} = \gamma_{M1} = 1.0$, f_y from EN 1993-1-1 Table 3.1, buckling lengths $1.0 \cdot L$ about both axes and in torsion. The assessment finds the principal axes from the section ($\alpha = 45^\circ$) and checks buckling about u and v; the buckling curve follows from the section (angle → b).

4. Results comparison

Quantity	Formula	Hand solution	FEM / assessment	Difference
Support reactions, axial force and shortening (FEM, unrounded)				
R_z base, strut 1 [kN] (CO1)	$1.35 \cdot G + 1.5 \cdot Q$	141.0000	141.00000000000000	0
R_z base, strut 2 [kN] (CO1)	$1.35 \cdot G + 1.5 \cdot Q$	141.0000	141.00000000000000	0
N_{Ed} , strut 1 [kN]	equilibrium	141.0000	141.00000000000000	0
N_{Ed} , strut 2 [kN]	equilibrium	141.0000	141.00000000000000	0
Shortening, strut 1, LC1 [mm]	$G \cdot L / (EA)$	0.11932	0.119324668108	≈ 0
Shortening, strut 2, LC1 [mm]	$G \cdot L / (EA)$	0.37289	0.372889587837	0
Section (assessment working, 1–2 decimals)				
$N_{Rk} = A \cdot f_y$ [kN]	class 3, gross area	450.152	450.15	2×10^{-3} (rounding)
i_0 [mm]	$\sqrt{(i_u^2 + i_v^2 + y_0^2 + z_0^2)}$	54.065	54.1	4×10^{-2} (rounding)
$N_{cr,T}$ [kN]	$G \cdot I_t / i_0^2$	1755.06	1755.06	2×10^{-3} (rounding)
Strut 1 — L = 0.8 m (assessment working, 1–3 decimals)				
$\bar{\lambda}_u$	$L / i_u / \lambda_1$	0.2227	0.223	3×10^{-4} (rounding)
χ_u	curve b	0.9920	0.992	5×10^{-5} (rounding)
$\bar{\lambda}_v$	$L / i_v / \lambda_1$	0.4364	0.436	4×10^{-4} (rounding)
χ_v	curve b	0.9114	0.911	4×10^{-4} (rounding)
$N_{cr,u}$ [kN]	$\pi^2 \cdot E \cdot I_u / L^2$	9077.77	9077.77	2×10^{-3} (rounding)
$N_{cr,TF}$ [kN]	lower root	1624.42	1624.42	5×10^{-3} (rounding)
$\bar{\lambda}_T$	$\sqrt{(N_{Rk} / N_{cr,TF})}$	0.5264	0.526	4×10^{-4} (rounding)
χ_T	curve b	0.8723	0.872	3×10^{-4} (rounding)
$N_{b,T,Rd}$ [kN]	$\chi_T \cdot A \cdot f_y / \gamma_{M1}$	392.68	392.68	1×10^{-3} (rounding)
Strut 2 — L = 2.5 m (assessment working, 1–3 decimals)				
$\bar{\lambda}_u$	$L / i_u / \lambda_1$	0.6959	0.696	1×10^{-4} (rounding)
χ_u	curve b	0.7860	0.786	3×10^{-5} (rounding)
$\bar{\lambda}_v$	$L / i_v / \lambda_1$	1.3636	1.364	4×10^{-4} (rounding)
χ_v	curve b	0.3974	0.397	4×10^{-4} (rounding)
$N_{cr,u}$ [kN]	$\pi^2 \cdot E \cdot I_u / L^2$	929.56	929.56	3×10^{-3} (rounding)
$N_{cr,TF}$ [kN]	lower root	734.51	734.51	4×10^{-3} (rounding)
$\bar{\lambda}_T$	$\sqrt{(N_{Rk} / N_{cr,TF})}$	0.7829	0.783	1×10^{-4} (rounding)

χ_T	curve b	0.7350	0.735	3×10^{-5} (rounding)
$N_{b,T,Rd}$ [kN]	$\chi_T \cdot A \cdot f_y / \gamma_{M1}$	330.85	330.85	1×10^{-3} (rounding)
Utilizations (assessment, unrounded)				
Compression 6.2.4 (both struts)	$N / (A \cdot f_y)$	0.3132	0.313227253783439	0
Strut 1, flexural about u	$N / N_{b,u,Rd}$	0.3158	0.315768086608243	0
Strut 1, flexural about v	$N / N_{b,v,Rd}$	0.3437	0.343684279347336	0
Strut 1, flexural-torsional	$N / N_{b,T,Rd}$	0.3591	0.359072022269563	0
Strut 2, flexural about u	$N / N_{b,u,Rd}$	0.3985	0.398494495330335	0
Strut 2, flexural about v	$N / N_{b,v,Rd}$	0.7881	0.788099058161798	≈ 0
Strut 2, flexural-torsional	$N / N_{b,T,Rd}$	0.4262	0.426176304275575	≈ 0
Governing (overall)	max	0.7881	0.788099058161798	≈ 0

Forces in kN. The utilizations are unrounded and agree to floating-point round-off. The working the assessment prints (N_{Rk} , i_0 , the critical loads, $\bar{\lambda}$, χ , $N_{b,Rd}$) is shown to 1–3 decimals; those rows agree to the last printed digit, and the differences shown are that rounding, not a calculation error.

✓ All quantities match; governing check flexural buckling about v–v of strut 2, $\eta = 0.788$; strut 1 governed by flexural-torsional buckling, $\eta = 0.359$ **VERIFIED**

5. Final design: maximum design force

The largest design axial force each strut can carry is its lowest resistance over all checks, $N_{Rd,max} = \min(N_{c,Rd}, N_{b,u,Rd}, N_{b,v,Rd}, N_{b,T,Rd})$. By hand, from section 2 ($\gamma_{M0} = \gamma_{M1} = 1.0$, $N_{Ed} = 141$ kN):

Check	Resistance by hand [kN]	$\eta = N_{Ed}/N_{Rd}$
Strut 1 — L = 0.8 m		
cross-section, $N_{c,Rd}$	450.15	0.3132
flexural about u, $N_{b,u,Rd}$	446.53	0.3158
flexural about v, $N_{b,v,Rd}$	410.26	0.3437
flexural-torsional, $N_{b,T,Rd}$	392.68	0.3591
Strut 2 — L = 2.5 m		
cross-section, $N_{c,Rd}$	450.15	0.3132
flexural about u, $N_{b,u,Rd}$	353.83	0.3985
flexural about v, $N_{b,v,Rd}$	178.91	0.7881
flexural-torsional, $N_{b,T,Rd}$	330.85	0.4262

The assessment reports utilizations, not resistances. Its maximum design force is N_{Ed} divided by its largest utilization for the member, which is the governing resistance itself, unrounded:

Strut	Governing check	$N_{Rd,max}$ by hand [kN]	$N_{Rd,max} = N_{Ed}/\eta_{max}$, FEM [kN]	Difference
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Strut 1, L = 0.8 m	flexural-torsional	392.6789	392.678881269530	0
Strut 2, L = 2.5 m	flexural about v	178.9115	178.911519484461	6×10^{-14}

✓ **Maximum design force: strut 1 $N_{Rd,max} = 392.68$ kN (flexural-torsional buckling), strut 2**

$N_{Rd,max} = 178.91$ kN (flexural buckling about v); hand and FEM agree VERIFIED

Against $N_{Ed} = 141$ kN, strut 1 has a reserve of 64 % and strut 2 a reserve of 21 %. Without the §6.3.1.4 check, strut 1 would be credited with $N_{b,v,Rd} = 410.26$ kN, 4.5 % more than it can carry.

6. Conclusion

The FEM calculator's steel assessment reproduces the EN 1993-1-1 hand calculation of an open, singly symmetric section in compression. It classifies the angle by its own Table 5.2 rule, finds the principal axes and checks flexural buckling about u–u and v–v. It also runs the §6.3.1.4 check: it places the shear centre at the leg midlines, computes i_0 and $N_{cr,T}$, couples torsion with flexure about the axis of symmetry and finds $N_{cr,TF}$. Its resistances and utilizations match the hand calculation, including which mode governs each strut and the maximum design force that follows from it.

Scope: the load here acts through the centroid. A real angle bracing member is usually connected by one leg, and that eccentricity is handled by EN 1993-1-1 Annex BB.1.2 (effective slenderness of angle web members) or EN 1993-1-8 §3.10.3. Neither is part of this example.

The same comparison runs as part of the automated test suite on every test run:

`tests/verificationExamples.test.ts` solves the project file `calc_tests/verification/steel-angle-strut.json`, runs the steel assessment on CO1 and asserts the support reactions, axial forces, shortening, the class, the principal-axis buckling working, i_0 , $N_{cr,T}$, $N_{cr,u}$, $N_{cr,TF}$, every utilization and the maximum design force of each strut against the hand calculation above, recomputed from the section and material stored in the file.