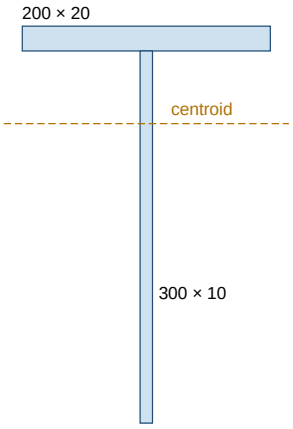
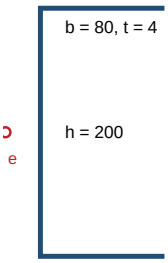
Verification — Cross-Section Properties

Cross-section calculator (femonline.eu/section/) checked against hand calculations, closed-form torsion theory and printed catalogue values

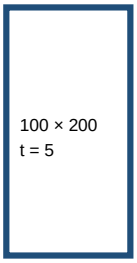
Sections	Solid rectangle; tee from two touching plates; thin-walled channel; closed thin-walled box; rolled IPE 300
Verification method	Parallel-axis theorem and plastic neutral axis by hand; St Venant series for a rectangle; Vlasov thin-walled theory; Bredt's formula; the published EN 10365 catalogue; EN 1993-1-1 Table 5.2
Quantities checked	A, centroid, second moments, elastic and plastic moduli, It, Iw, shear centre, shear area; cross-section class; class 4 effective section; shear flow across a cut; elastic stresses
Result	Outline properties, the shear flow and the class 4 effective section match the hand values to rounding. Finite-element values are within 0.25 % of the thin-walled closed forms for the channel, 1.6 % of Bredt for the box, and 1.6 % of the printed It and 1.4 % of the formula Iw for the IPE 300. The larger differences are explained in sections 4 and 5. Stresses read away from sharp corners are within 3 % of the closed forms
Reproduce	Open the sections in the app; the automated test `tests/sectionVerification.test.ts` repeats every comparison on each build



Tee, two plates



Channel, mid-line



Box, mid-line

1. Solid rectangle, b = 100, h = 50 mm

$A = b \cdot h$ · $I_y = b \cdot h^3/12$ · $I_z = h \cdot b^3/12$ · $W_{el,y} = b \cdot h^2/6$ · $W_{pl,y} = b \cdot h^2/4$
$I_t = (b \cdot h^3/3) \cdot [1 - (192/\pi^5)(h/b)\, \Sigma_{n\ odd}\, \tanh(n\pi b/2h)/n^5]$ · $A_{s,z} = 5/6 \cdot A$ (energy-equivalent shear area, ν = 0)

Quantity	Hand	Calculator	Unit	Difference
A	5 000	5 000	mm²	$< 10^{-6}$ %
Iy	1 041 666.7	1 041 666.7	mm⁴	$< 10^{-6}$ %
Iz	4 166 666.7	4 166 666.7	mm⁴	$< 10^{-6}$ %
Wel,y	41 666.7	41 666.7	mm³	$< 10^{-6}$ %
Wpl,y	62 500	62 500	mm³	$< 10^{-6}$ %
It	2 858 521	2 858 524	mm⁴	$1.0 \cdot 10^{-4}$ %
Asz	4 166.7	4 166.7	mm²	$3.8 \cdot 10^{-5}$ %

2. Tee from two plates: flange 200 × 20 on web 300 × 10 mm

> z measured from the bottom of the web. A = 4000 + 3000 = 7000 mm²
> z_c = (4000·310 + 3000·150)/7000 = 241.43 mm
> I_y = 200·20³/12 + 4000·(310 − 241.43)² + 10·300³/12 + 3000·(150 − 241.43)² = 66.52·10⁶ mm⁴
> Plastic neutral axis: half the area, 3500 mm², lies above it, so it is d = 3500/200 = 17.5 mm into the flange
> $W_{pl,y}$ = 200·17.5·8.75 + 200·2.5·1.25 + 3000·(2.5 + 150) = 488 750 mm³

Quantity	Hand	Calculator	Unit	Difference
A	7 000	7 000	mm²	< 10^{-6} %
zc	241.429	241.429	mm	< 10^{-6} %
Iy	$6.6519 \cdot 10^{7}$	$6.6519 \cdot 10^{7}$	mm⁴	< 10^{-6} %
Iz	$1.3358 \cdot 10^{7}$	$1.3358 \cdot 10^{7}$	mm⁴	< 10^{-6} %
Wel,y,top	846 606.1	846 606.1	mm³	< 10^{-6} %
Wel,y,bot	275 522.7	275 522.7	mm³	< 10^{-6} %
Wpl,y	488 750	488 750	mm³	< 10^{-6} %

The plates touch, so the calculator merges them into one fully connected outline and says so on screen and in its report. That assumes a continuous connection between them.

3. Thin-walled channel, h = 200, b = 80, t = 4 mm (mid-line dimensions)

> I_t = (h + 2b)·t³/3 = 7680 mm⁴ · e = 3b²/(h + 6b) = 28.24 mm behind the web mid-line
> I_w = t·b³·h²·(3b + 2h) / [12·(6b + h)] = 6.425·10⁹ mm⁶

Quantity	Closed form	Calculator	Unit	Difference
A	1 440	1 440	mm²	< 10^{-6} %
It, mid-line	7 680	7 680	mm⁴	< 10^{-6} %
Iw, mid-line	$6.4251 \cdot 10^{9}$	$6.4251 \cdot 10^{9}$	mm⁶	< 10^{-6} %
e, mid-line	28.235	28.235	mm	< 10^{-6} %
It, FE	7 680	7 665.7	mm⁴	0.19 %
Iw, FE	$6.4251 \cdot 10^{9}$	$6.4394 \cdot 10^{9}$	mm⁶	0.22 %
e, FE	28.235	28.177	mm	0.21 %

The calculator's two routes are compared separately. Its mid-line model is the same theory as the closed forms, so it agrees to rounding. Its finite-element solve works on the real outline with thickness, so a difference of a few tenths of a percent is expected here and is not an error in either.

4. Closed box, 100 × 200 mm, t = 5 mm (mid-line dimensions)

> Bredt: $I_t = 4 \cdot A_m^2 \cdot t / \Sigma s$ = 4·20 000²·5 / 600 = 13.33·10⁶ mm⁴

Quantity	Bredt	Calculator	Unit	Difference
It, FE	$1.3333 \cdot 10^{7}$	$1.3547 \cdot 10^{7}$	mm⁴	1.60 %

Bredt's formula assumes thin walls with uniform shear stress across the thickness. The finite-element result is the higher one because it also includes the torsional stiffness of the walls themselves and the outer corners. A difference of this size is expected at b/t = 20.

5. Rolled IPE 300 against the printed catalogue

> Reference: EN 10365 values as printed in the ArcelorMittal and Peiner catalogues, rounded to three significant figures (the printed table is the source of differences up to about 0.1 % in A and the moduli).
> The printed table has no I_w, so it is compared with $I_w = t_f \cdot b^3 \cdot (h - t_f)^2/24$ for a doubly symmetric I, which ignores the fillets, so the finite-element value need not equal it.

Quantity	Reference	Calculator	Unit	Difference
A	5 380	5 382.3	mm²	0.04 %
Iy	$8.3560 \cdot 10^7$	$8.3581 \cdot 10^7$	mm⁴	0.03 %
Iz	6 040 000	6 037 900.4	mm⁴	0.03 %
Wel,y	557 000	557 209.7	mm³	0.04 %
Wpl,y	628 000	628 507.8	mm³	0.08 %
It, FE	201 000	197 776.1	mm⁴	1.60 %
Iw, FE	$1.2593 \cdot 10^{11}$	$1.2425 \cdot 10^{11}$	mm⁶	1.34 %

Printed I_t values come from an approximate formula rather than a torsion solve, so the 1.6 % in I_t is a difference between methods. It is not shown to be an error in the printed value or in the solve.

6. Cross-section class of the IPE 300 in compression (EN 1993-1-1 Table 5.2)

> Web: c = 300 − 2·10.7 − 2·15 = 248.6 mm, c/t = 248.6/7.1 = 35.01
> S355: ε = √(235/355) = 0.814, 42ε = 34.17 < 35.01, so **class 4** · S235: ε = 1, 33 < 35.01 ≤ 38, so **class 2**

Quantity	Hand	Calculator	Unit	Difference
web c/t	35.014	35.014		$< 10^{-6}$ %
ε, S355	0.814	0.814		$< 10^{-6}$ %
limit 42ε, S355	34.172	34.172		$< 10^{-6}$ %

The calculator returns class 4 in S355 and class 2 in S235, matching the hand result.

7. Shear flow at the flange-to-web joint of the tee (V_z = 250 kN)

> $S = A_f \cdot (z_f - z_c) = 4000 \cdot (310 - 241.43) = 274\,286$ mm³ · $q = V \cdot S / I_y = 250\,000 \cdot 274\,286 / 66.52 \cdot 10^6 = 1030.9$ N/mm
> Mean stress in the 10 mm web: q/t = 103.1 MPa · Weld stress with Σa = 8 mm: q/Σa = 128.9 MPa

Quantity	Hand	Calculator	Unit	Difference
S (flange)	274 285.7	274 285.7	mm³	$< 10^{-6}$ %
q	1 030.9	1 030.9	N/mm	$< 10^{-6}$ %
τ in the web	103.085	103.085	MPa	$< 10^{-6}$ %
τ‖, Σa = 8	128.857	128.857	MPa	$< 10^{-6}$ %

The calculator uses the general form for unsymmetric sections, which reduces to V·S/I on principal axes; the automated tests also compare it with a principal-axis calculation on an unequal angle. The weld stress is the longitudinal stress only. It is not a weld check.

8. Elastic stresses

> Rectangle 100 × 50: $\sigma = N/A + M_y/W$ (N = 100 kN, M_y = 5 kNm) · $\tau = 1.5 \cdot V/A$ (V_z = 50 kN) · $\tau = T/(k_2 \cdot a \cdot b^2)$, k_2 = 0.246 (T = 5 kNm)
> Tee: $\sigma = M_y \cdot z/I_y$ at the top fibre (M_y = 100 kNm) · τ at the centroid = $V \cdot S/(I_y \cdot t)$, V_z = 250 kN, read at that point

Quantity	Closed form	Calculator	Unit	Difference
σ, N + My, rectangle	140.000	140.000	MPa	$< 10^{-6}$ %
τ, Vz, rectangle	15.000	15.006	MPa	0.04 %
τ, T, rectangle	81.301	81.342	MPa	0.05 %
τ, Vz, tee web at centroid	109.532	109.532	MPa	$1.0 \cdot 10^{-4}$ %
σ, My, tee top fibre	118.119	118.119	MPa	$< 10^{-6}$ %

Normal stress is exact. Shear stress is read from the finite-element field on the fine mesh. The automated tests also confirm that on these sections the shear field adds back up to the applied forces and torque with the right signs, and that a force through the shear centre gives no torque.

Sharp inner corners. At a re-entrant corner the elastic torsion stress has no finite value. On an angle from two plates under T = 0.1 kNm, the peak reads 24.7, 31.4 and 37.4 MPa on the coarse, normal and fine meshes, while a point away from the corner stays at 16.1 MPa, matching $T \cdot t/I_t$. The calculator therefore reads the peak on a second mesh level and warns when it changes by more than 5 %. These plots show elastic St Venant stresses: warping stresses from restrained torsion are not included.

9. Class 4 effective section: plate girder, flanges 300 × 20, web 1000 × 8, S355 (EN 1993-1-5)

> ε = 0.814, web c/t = 125. **Compression**, ψ = 1, k_σ = 4: $\bar{\lambda}_p = 125/(28.4 \cdot \varepsilon \cdot 2) = 2.705$, $\rho = (\bar{\lambda}_p - 0.22)/\bar{\lambda}_p^2 = 0.340$, $A_{eff} = A - (1 - \rho) \cdot 1000 \cdot 8$
> **Bending**, ψ = −1, k_σ = 23.88: $\bar{\lambda}_p = 1.107$, $\rho = (\bar{\lambda}_p - 0.11)/\bar{\lambda}_p^2 = 0.814$; b_c = 500, $b_e = \rho \cdot 500$; the hole runs from $0.4 \cdot b_e$ below the top of the web to $0.6 \cdot b_e$ above mid-height. W_{eff} from the second moment of the remaining area about its shifted axis, to the farther fibre

Quantity	Hand	Calculator	Unit	Difference
ρ web, compression	0.340	0.340		$< 10^{-6}$ %
Aeff	14 717.1	14 717.1	mm²	$< 10^{-6}$ %
ρ web, bending	0.814	0.814		$< 10^{-6}$ %
Weff,y	7 006 530.7	7 006 530.7	mm³	$< 10^{-6}$ %

The automated tests also compare A_{eff} and W_{eff} with the FEM calculator's own effective-section module on a welded I, a hollow section and an angle. The effective section gives properties: it is not a resistance or member check.

10. Limits of this verification

Closed-form references exist only for these simple shapes. For a free-form section the calculator gives its mesh check instead, which shows how much It, Iw, the shear areas and the shear centre change between two mesh levels. That measures how far the result depends on the mesh, but two agreeing meshes do not prove a result correct. The classification does not read plates from general polygons: it names such shapes as not classified when they look thin-walled.